RIEMANN-ROCH THEOREMS FOR FLAT BUNDLES

SPENCER BLOCH AND HÉLÈNE ESNAULT

ABSTRACT. Let $f: U \to \mathrm{Spec}\,(K)$ be a smooth open curve over a field $K \supset k$, where k is an algebraically closed field of characteristic 0. Let $\nabla : L \to L \otimes \Omega^1_{U/k}$ be a (possibly irregular) absolutely integrable connection on a line bundle L. In the lecture, we explained how to use the polar part of the connection ∇ to determine the determinant of de Rham cohomology with its Gauß-Manin connection $\Big(\det Rf_*(L \otimes \Omega^1_{U/K}),\ \det \nabla_{GM}\Big)$ in $\Omega^1_K/d\log K^*$ modulo torsion. We showed that this expression, which is obtained as the trace of a certain cup-product, might be also understood as a sum of local contributions in the singularities of the connection, after chosing a rational section of the relative 1-forms. We showed how this leads naturally to consider a generalized twisted jacobian in the higher rank case, and stated a conjecture on the general shape of the determinant of the relative de Rham cohomology.

REFERENCES

[1] Bloch, S.; Esnault, H.: A Riemann-Roch theorem for flat bundles, with values in the algebraic Chern-Simon theory, preprint, (to appear in the Annals of Mathematics), 1998, 47 pages.
[2] Bloch, S., Esnault, H.: Gauß-Manin determinants for rank 1 irregular connections on curves.

DEPT. OF MATHEMATICS, UNIVERSITY OF CHICAGO, CHICAGO, IL 60637, USA
E-mail address: bloch@math.uchicago.edu

MATHEMATIK, UNIVERSITÄT ESSEN, FB6, MATHEMATIK, 45117 ESSEN, GERMANY
E-mail address: esnault@uni-essen.de

Date: June 19, 1999.