# ON CLASS NUMBER OF QUADRATIC FUNCTION FIELDS

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by

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Abstract

A table of class number of quadratic function field $K = \mathbb{F}_p(T, \sqrt{P})$ is presented for $p = 3$, $3 \leq \deg P \leq 8$; $p = 5$, $3 \leq \deg P \leq 5$; $p = 7,11$, $3 \leq \deg P \leq 4$ and $P = P(T)$ is an irreducible polynomial in $\mathbb{F}_p[T]$. The data are intended to varify the Friedman–Washington conjecture. A version of a S. Chowla conjecture for real quadratic function field is proved.

## 1. Introduction

Researching on ideal class number of quadratic function fields was initiated by E. Artin's thesis [1] at 1924. By using Gauss classical method, Artin worked on binary quadratic forms over $\mathbb{F}_p[T]$ $(p \geq 3)$ and gave the class number formula of quadratic function fields. The problem on divisor class number of quadratic function fields (elliptic and hyperelliptic curves) is a special case of arithmatic theory of general algebraic curves on which there are huge contributions. Among them, for instance, MacRae [7], Madan, Leitzel and Queen [6, 9, 10] find several kind of quadratic function fields with small class number, Hayes [5] establishes a relation of ideal class numbers between real and imaginary quadratic function fields as an analogue of D. Zagier's formula for quadratic number fields, Datskovsky and Wright [2] calculate the average value of ideal 3–class number of quadratic function fields, Friedman and Washington [4] raise a conjecture on divisor class number of quadratic function fields as an analogue of Cohen–Lenstra conjecture for number field case.

Concerning on calculating data on class number of quadratic function fields, there is, by authors knowledge, only Artin's small table [1] for $K = \mathbb{F}_p(T,\sqrt{f(T)})$ with $p = 3$, $\deg f = 3,4$; $p = 5,7$, $\deg f = 3$. In this paper we present further data on class number of $K$ for all irreducible polynomials $f(T)$ in $\mathbb{F}_p[T]$ with $p = 3$, $5 \leq \deg f \leq 8$, $p = 5$, $\deg f = 4,5$; $p = 7$, $\text{def}\, f = 4$ and $p = 11$, $\deg f = 3,4$ by using MACINTOSH SE Computer at Max–Planck–Institut für Mathematik in Bonn. We thank MPI für Mathematik for its warmful hospitality and offering facilities. We also thank Eduardo Friedman for introducing their conjecture and useful discussions.

## 2. Computing method

### 2.1. class number formula

Let p be an odd prime number, P(T) an irreducible polynomial in $\mathbb{F}_p[T]$ , $k = \mathbb{F}_p[T]$ , $K = k(\sqrt{P})$ , $\deg P = d \geq 3$ , sgn P the coefficient of leading term of P . Without lossing of generality we assume sgn P = 1 or g where g is a primitive element of $\mathbb{F}_p$ .

According to Artin [1], K is called real if $2 | d$ and sgn P = 1 (Namely, $\sqrt{P} \in k_\infty$ , where $k_\infty$ is the completion of k at the infinite divisor $\infty = (\frac{1}{T}))$ . Otherwise K is called imaginary.

Let H(P) and h(P) denote the class number of $K = k(\sqrt{P})$ for divisor and ideal class group respectively. It is well–known that ( [7] p. 247)

(1) For real K ,

$$H(P) = h(P)R(P)$$

where $R = R(P) = \log_p |\varepsilon|_{\mathfrak{p}} = V_{\mathfrak{p}}(\varepsilon)$ is the regulator of K , $\varepsilon$ is the fundamental unit of K and $\mathfrak{p}$ is arbitrary infinite prime of K .

(2) For imaginary K ,

$$H(P) = \begin{cases} h(P) , & \text{if } 2 \nmid d \\ h(P)/2 , & \text{if } 2 \mid d \text{ and } \operatorname{sgn} P = g \ . \end{cases}$$

We calculated h(P) for imaginary K and H(P) = h(P)R for real K without computing fundamental unit $\varepsilon$ and regular R. Our computation is based on the class number formula presented by Artin [1].

From now on, let $P = P(T)$ be a monic irreducible polynomial in $\mathbb{F}_p[r]$. For each $Q = Q(T) \in \mathbb{F}_p[r]$, $P \nmid Q$, the Legendre symbol means

$$\left[\frac{Q}{P}\right] = \begin{cases} 1, & \text{if } Q \text{ is a square mod } P \\ -1, & \text{otherwise} \end{cases}.$$

And the Jacobi symbol is also defined in usual way. We have the reciprocity law [1] : for monic polynomials M and N in $\mathbb{F}_p[T]$, $(M,N) = 1$, $|M| = p^{\deg M}$,

$$\left[\frac{M}{N}\right]\left[\frac{N}{M}\right] = (-1)^{\frac{|M|-1}{2}\cdot\frac{|N|-1}{2}} = (-1)^{\frac{p-1}{2}\deg M \cdot \deg N}.$$

Let

$$\sigma_i(P) = \sum_{\substack{A \in \mathbb{F}_p[T] \\ \text{monic} \\ \deg A = i}} \left[\frac{P}{A}\right] \qquad (1)$$

The Artin's class number formulas are as following

(1) If $\deg P = 2m$, the class number of real field $K = k(\sqrt{P})$ is

$$H(P) = h(P)R = -\sum_{i=1}^{2m-1} i\sigma_i(P)\,.$$

And the class number for imaginary field $K = k(\sqrt{gP})$ is

$$h(gP) = \sum_{i=0}^{2m-1} \sigma_i(gP)\,.$$

The quadratic reciprocity law implies that ( [1], p. 210)

$$\sigma_i(gP) = (-1)^i \sigma_i(P)\,, \tag{2}$$

thus we need to compute $\sigma_i(P)$ $(0 \le i \le 2m-1)$ only. Moreover, we have ( [1], p. 227)

$$\sigma_{2m-i} = p^{m-i}[-\sigma_{i-1} + (p-1)(\sigma_{i-2} + \ldots + \sigma_1 + \sigma_0)] \quad (2 \le i \le 2m)$$

and $\sigma_0 = 1$ , $\sigma_{2m-1} = -p^{m-1}$ . Therefore it is enough to compute $\sigma_i(P)$ for $2 \le i \le m-1$ only.

(2) If $\deg P = 2m+1$ , the class number for imaginary field $k(\sqrt{P})$ and $k(\sqrt{gP})$ are

$$h(P) = (1+p^m) + (1+p^{m-1})\,\sigma_1(P) + \ldots + (1+p)\,\sigma_{m-1}(P) + \sigma_m(P)$$

$$h(gP) = (1+p^m) + (1+p^{m-1})\,\sigma_1(gP) + \ldots + (1+p)\,\sigma_{m-1}(gP) + \sigma_m(gP)$$

From formula (2) we know that it is enough to compute $\sigma_i(P)(1 \le i \le m)$ only.

2.2. computation of $\sigma_i(P)$

For each $d \geq 3$ we choose a fixed primitive polynomial $P_0(T)$ in $\mathbb{F}_p[T]$, $\deg P_0 = d$. Let $\alpha$ be a root of $P_0(T)$. Compute the multiplication law (index–vector correspondance):

$$\alpha^i = a_{d-1}\,\alpha^{d-1} + \dots + a_1\alpha + a_0 = (a_{d-1}, \cdots, a_1, a_0) \qquad (3)$$

$$(a_i \in \mathbb{F}_p\,,\ 0 \leq i \leq p^d - 2)\,.$$

For each polynomial $A(T) = T^i + c_1T^{i-1} + \dots + c_i \in \mathbb{F}_p[T]$, find the value of $t$ in the above multiplicative law (3) such that $\alpha^i + c_1\alpha^{i-1} + \dots + c_i = \alpha^t$. Then

$$\left[\frac{A}{P_0}\right] = (-1)^t\,.$$

In this way we can compute

$$\tau_i(P_0) = \sum_{\substack{A \in \mathbb{F}_p[r] \\ \text{monic} \\ \deg A = i}} \left[\frac{A}{P_0}\right]$$

and $\sigma_i(P_0) = (-1)^{\frac{p-1}{2} \cdot i \cdot d}\,\tau_i(P_0)$ by reciprocity law.

Now suppose that $P = P(T)$ is arbitrary monic irreducible polynomial in $\mathbb{F}_p[T]$ with degree $d$ and $\beta = \alpha^n$ is a root of $P[T]$. For each $A = A(T) = T^i + c_1T^{i-1} + \ldots + c_i \in \mathbb{F}_p[T]$, let

$$\beta^i + c_1\beta^{i-1} + \ldots + c_i = \alpha^{ni} + c_1\alpha^{n(i-1)} + \ldots + c_i = \alpha^s$$

and compute the value of $s$ by using multiplicative law (3). It is easy to see that

$$\left[\frac{A}{P}\right] = (-1)^s .$$

Therefore we can compute $h(P)$ and $h(gP)$ for all monic irreducible polynomials $P$ with degree $d$ by using multiplicative law (3) for just one polynomial $P_0$.

2.3. Specification of tables

A polynomial $P(T) = T^d + a_1T^{d-1} + \ldots + a_d (a_i \in \mathbb{F}_p)$ is denoted by $(1a_1 \ldots a_d)$ in the tables and listed with lexicography order.

For each $a \in \mathbb{F}_p$, let $Q(T) = P(T + a)$. It is easy to see that $h(Q) = h(P)$, $H(Q) = H(P)$. Therefore we list the "minimal" (by lexicograph order) polynomial from each class $\{P(T + a) | a \in \mathbb{F}_p\}$. Particularly, if $p \mid d$ we list the irreducible polynomials with $a_1 = 0$. The polynomial $P(T) = (1a_1 \ldots a_d)$ with (*) means that $P(T + a) = P(T)$ for all $a \in \mathbb{F}_p$.

3. On the Friedman–Washington conjecture

Let $\ell$ be an odd prime number, $\ell \neq p$, H a finite abelian $\ell$ – group, $\gamma = \gamma(H) = \dim_{\mathbb{Z}/\ell\mathbb{Z}}(H/\ell H)$ the $\ell$ – rank of H, $k = \mathbb{F}_p(T)$, N(g) the number of quadratic function fields $K = k(\sqrt{f(T)})$ with $\deg f = 2g + 1$ and $2g + 2$ (so the genus of K is g), N(H,g) the number of this kind of K such that the $\ell$ – primary part of divisor class group of K is isomorphic to H. Thus

$$N(g) = 2(M(2g + 1) + M(2g + 2))$$

where $M(n) = \frac{1}{n} \sum_{d|n} \mu(d)p^{n/d}$ is the number of monic irreducible polynomials in $\mathbb{F}_p[T]$.

Friedman and Washington raise a conjecture in [4] that the ratio N(H,g)/N(g) is expected to close with

$$\mu(H,g) = \frac{1}{|Aut(H)|} \left[ \prod_{j=1}^{2g} (1 - \ell^{-j}) \right] \prod_{j=2g-r+1}^{2g} (1 - \ell^{-j}) .$$

for big g.

The following data for p = 3 and g = 2,3 taken from our tables is intended to check this conjecture (Note, $C_n^{\ell}$ denotes the product of $\ell$ copies of cyclic group with order n, $N(C_5^2 + C_{25}, g) = N(C_5^2, g) + N(C_{25}, g)$, $\mu(C_5^2 + C_{25}, g) = \mu(C_5^2, g) + \mu(C_{25}, g))$.

(I) $p = 3$ , $g = 2$ , $N(2) = 2(48 + 116) = 328$

| H | $C_5$ | $C_7$ |
|---|---|---|
| N(H,2) | 84 | 72 |
| N(H,2)/N(2) | 0.2561 | 0.2195 |
| $\mu$(H,2) | 0.1899 | 0.1394 |

(II) $p = 3$ , $g = 3$ , $N(3) = 2(312 + 810) = 2244$

| H | $C_5$ | $C_7$ | $C_{11}$ | $C_{13}$ | $C_{17}$ | $C_5^2+C_{25}$ |
|---|---|---|---|---|---|---|
| N(H,3) | 432 | 276 | 210 | 150 | 132 | 72 |
| N(H,3)/N(3) | 0.1925 | 0.1230 | 0.0936 | 0.0668 | 0.0588 | 0.0321 |
| $\mu$(H,3) | 0.1901 | 0.1395 | 0.0901 | 0.0764 | 0.0586 | 0.0396 |

The situation for the case $g = 3$ is better than the case $g = 2$ eventhough $g = 3$ is still too small and our average is taken on irreducible polynomials only.

4. A version of a S. Chowla's conjecture

Let p , g be odd prime numbers and $p = 4q^2 + 1$ . From Brauer–Siegel Theorem we know that there are only finitely many of such p such that the class number h(p) of real quadratic field $\mathbb{Q}(\sqrt{p})$ is one, S. Chowla's conjecture says that:

$$h(p) = 1 \text{ for such } p \Leftrightarrow (p,q) = (37,3)\ ,\ (101,5)\ ,\ (197,7) \text{ and } (677,13)\ .$$

The conjecture has not been proved without a suitable Riemann hypothesis (see note at the end of [8]). Now we can prove a version of this conjecture for real quadratic function field as an application of our tables.

Theorem. Suppose $2 \nmid q$ , $k = \mathbb{F}_q(T)$ , $P = P(T)$ an irreducible polynomial in $\mathbb{F}_q[T]$ and $P(T) = A(T)^2 + a$ where $a \in \mathbb{F}_q$ , A(T) is monic polynomial in $\mathbb{F}_q[T]$ and $D = \deg A \geq 2$ . For this kind 0f P(T) , there are exact six real quadratic function fields $K = k(\sqrt{P})$ such that the ideal class number h(P) of K is one. They are

$$
\begin{array}{lll}
q = 3\ , & (P,A) = & (T^6 + T^4 + T^3 + T^2 - T - 1\ ,\ T^3 - T - 1) \\
 & & (T^6 + T^4 - T^3 + T^2 + T - 1\ ,\ T^3 - T + 1) \\
 & & (T^4 - T^2 - 1\ ,\ T^2 + 1) \\
 & & (T^4 - T^3 - T^2 + T - 1\ ,\ T^2 + T - 1) \\
 & & (T^4 + T^3 - T^2 - T - 1\ ,\ T^2 - T - 1) \\
q = 5\ , & (P,A) = & (T^4 + 2\ ,\ T^2)
\end{array}
$$

(We exclude the case $D = 1$ since the genus of $K = k(\sqrt{P})$ is $D - 1 = 0$ and $h(P) = 0$ is always one in this case.)

Proof. Since $\varepsilon = A + \sqrt{A^2 + a}$ is the fundamental unit of K and the regulator of K is $R = \deg A = D$, we need to determine all $K = k(\sqrt{P})$ with $H(P) = Rh(P) = D$.

Following argument is taken from [9], p. 424. Let $\bar{K}/\bar{k}$ be the constant extension of K/ k of degree $2g_k - 1$. The genus of $\bar{K}$ is $g = g_k = D - 1$. Applying the Weil theorem to $\bar{K}/\bar{k}$ we obtain

$$\bar{N}_1 \geq q^{2g-1} + 1 - 2g \cdot q^{(2g-1)/2} = q^{2D-3} + 1 - 2(D-1)q^{(2D-3)/2}$$

where $\bar{N}_1$ denotes the number of primes with degree one in $\bar{K}$.

A prime of degree e of K decomposes in $\bar{K}$ as product of (e , 2g – 1) primes of degree e/(e , 2g – 1) (see [3], p. 164). Therefore, by choosing e = 2g – 1 , K has at least $(q^{2D-3} + 1 - 2(D-1)q^{(2D-3)/2} / (2g-1)$ integral divisors of degree $2g - 1 = 2D - 3$. On the other hand, the Riemann–Roch theorem implies that a divisor class of degree 2g – 1 has dimension g . There are precisely $(q^g - 1)/(q-1)$ integral divisors in such a class ([2], p. 64). Therefore

$$\frac{H(P)(q^{D-1} - 1)}{q-1} \geq \frac{q^{2D-3} + 1 - 2(D-1)q^{(2D-3)/2}}{2D-3}$$

Namely,

$$h(P) \geq \frac{(q-1)(q^{2D-3} + 1 - 2(D-1)q^{(2D-3)/2}}{D(2D-3)(q^{D-1} - 1)} \qquad (4)$$

It is easy to see that the right side of (4) is $> 1$ if $q \geq 7$, $D \geq 2$; $q = 5$, $D \geq 3$; or $q = 3$, $D \geq 5$. For $q = 5$, $D = 2$ ($d = \deg P = 2D = 4$) and $q = 3$, $D = 2,3,4$ ($d = \deg P = 4,6,8$), we get the six fields in theorem by checking $H(P) = Rh(P) = D$ in our tables.

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# TABLES ON CLASS NUMBER OF QUADRATIC FUNCTION FIELDS

p=3 d=3

| | P(T) | h(p) | h(−p) |
|---|---|---|---|
| * | 1021 | 7 | 1 |
| * | 1022 | 1 | 7 |
| | 1102 | 3 | 5 |
| | 1201 | 5 | 3 |

p=3 d=4

| P(T) | H(p) | h(−p) |
|---|---|---|
| 10012 | 4 | 8 |
| 10022 | 4 | 8 |
| 10102 | 6 | 4 |
| 10111 | 6 | 4 |
| 10121 | 6 | 4 |
| 10202 | 2 | 12 |

p=3 d=5

| P(T) | h(p) | h(−p) |
|---|---|---|
| 100021 | 29 | 5 |
| 100022 | 5 | 29 |
| 100112 | 5 | 13 |
| 100211 | 13 | 5 |
| 101011 | 27 | 3 |
| 101012 | 3 | 27 |
| 101102 | 9 | 17 |
| 101122 | 7 | 15 |
| 101201 | 17 | 9 |
| 101221 | 15 | 7 |
| 102101 | 9 | 17 |
| 102112 | 11 | 19 |
| 102122 | 7 | 15 |
| 102202 | 17 | 9 |
| 102211 | 19 | 11 |
| 102221 | 15 | 7 |

p=3 d=6

| | P(T) | H(p) | h(−p) |
|---|---|---|---|
| | 1000012 | 13 | 26 |
| | 1000022 | 13 | 26 |
| | 1000111 | 21 | 10 |
| | 1000121 | 21 | 10 |
| | 1000201 | 35 | 6 |
| | 1010201 | 15 | 30 |
| | 1011001 | 21 | 10 |
| * | 1011122 | 3 | 38 |
| | 1012001 | 21 | 10 |
| * | 1012112 | 3 | 38 |
| | 1020001 | 35 | 6 |
| | 1020101 | 15 | 30 |
| | 1020112 | 9 | 18 |
| | 1020122 | 9 | 18 |
| | 1100002 | 13 | 26 |
| | 1100012 | 9 | 18 |
| | 1100111 | 23 | 14 |
| | 1101002 | 13 | 26 |
| | 1101011 | 15 | 30 |
| | 1101101 | 21 | 10 |
| | 1101112 | 23 | 14 |
| | 1101212 | 7 | 46 |
| | 1102001 | 13 | 26 |
| | 1102111 | 23 | 14 |
| | 1102121 | 19 | 6 |
| | 1102201 | 35 | 6 |
| | 1102202 | 5 | 42 |
| | 1200002 | 13 | 26 |
| | 1200022 | 9 | 18 |
| | 1200121 | 23 | 14 |
| | 1201001 | 13 | 26 |
| | 1201111 | 19 | 6 |
| | 1201121 | 23 | 14 |
| | 1201201 | 35 | 6 |
| | 1201202 | 5 | 42 |
| | 1202002 | 13 | 26 |
| | 1202021 | 15 | 30 |
| | 1202101 | 21 | 10 |
| | 1202122 | 23 | 14 |
| | 1202222 | 7 | 46 |

p=3 d=7

| P(T) | h(p) | h(−p) | P(T) | h(p) | h(−p) |
|---|---|---|---|---|---|
| 10000102 | 27 | 53 | 10101202 | 61 | 35 |
| 10000121 | 21 | 59 | 10101211 | 43 | 21 |
| 10000201 | 53 | 27 | 10102102 | 25 | 39 |
| 10000222 | 59 | 21 | 10102201 | 39 | 25 |
| 10001011 | 87 | 9 | 10110022 | 23 | 41 |
| 10001012 | 9 | 87 | 10110101 | 41 | 23 |
| 10001102 | 15 | 33 | 10110211 | 97 | 15 |
| 10001111 | 33 | 63 | 10111001 | 19 | 29 |
| 10001201 | 33 | 15 | 10111102 | 55 | 25 |
| 10001212 | 63 | 33 | 10111121 | 45 | 19 |
| 10002112 | 21 | 43 | 10111201 | 99 | 13 |
| 10002122 | 21 | 43 | 10112002 | 23 | 41 |
| 10002211 | 43 | 21 | 10112012 | 31 | 49 |
| 10002221 | 43 | 21 | 10112021 | 31 | 65 |
| 10010122 | 45 | 19 | 10112111 | 37 | 27 |
| 10010222 | 9 | 87 | 10112122 | 53 | 27 |
| 10011002 | 35 | 61 | 10120021 | 41 | 23 |
| 10011101 | 57 | 23 | 10120112 | 15 | 97 |
| 10011211 | 97 | 15 | 10120202 | 23 | 41 |
| 10012001 | 19 | 45 | 10121002 | 29 | 19 |
| 10012022 | 19 | 45 | 10121102 | 13 | 99 |
| 10012111 | 27 | 21 | 10121201 | 25 | 55 |
| 10012202 | 17 | 111 | 10121222 | 19 | 45 |
| 10020121 | 87 | 9 | 10122001 | 41 | 23 |
| 10020221 | 19 | 45 | 10122011 | 49 | 31 |
| 10021001 | 61 | 35 | 10122022 | 65 | 31 |
| 10021112 | 15 | 97 | 10122212 | 27 | 37 |
| 10021202 | 23 | 57 | 10122221 | 27 | 53 |
| 10022002 | 45 | 19 | 10200001 | 71 | 9 |
| 10022021 | 45 | 19 | 10200002 | 9 | 71 |
| 10022101 | 111 | 17 | 10200101 | 25 | 55 |
| 10022212 | 21 | 27 | 10200112 | 21 | 43 |
| 10100011 | 83 | 13 | 10200202 | 55 | 25 |
| 10100012 | 13 | 83 | 10200211 | 43 | 21 |
| 10100102 | 25 | 55 | 10201021 | 99 | 13 |
| 10100122 | 17 | 47 | 10201022 | 13 | 99 |
| 10100201 | 55 | 25 | 10201121 | 15 | 49 |
| 10100221 | 47 | 17 | 10201222 | 49 | 15 |
| 10101101 | 35 | 61 | 10202011 | 85 | 11 |
| 10101112 | 21 | 43 | 10202012 | 11 | 85 |

p=3 d=7

| P(T) | h(p) | h(−p) |
|---|---|---|
| 10210001 | 31 | 49 |
| 10210121 | 57 | 23 |
| 10210202 | 11 | 85 |
| 10211101 | 49 | 31 |
| 10211111 | 47 | 17 |
| 10211122 | 31 | 17 |
| 10211221 | 109 | 19 |
| 10212011 | 33 | 63 |
| 10212022 | 29 | 51 |
| 10212101 | 31 | 17 |
| 10212112 | 51 | 29 |
| 10212212 | 11 | 101 |
| 10220002 | 49 | 31 |
| 10220101 | 85 | 11 |
| 10220222 | 23 | 57 |
| 10221122 | 19 | 109 |
| 10221202 | 31 | 49 |
| 10221212 | 17 | 47 |
| 10221222 | 17 | 31 |
| 10222012 | 63 | 33 |
| 10222021 | 51 | 29 |
| 10222111 | 101 | 11 |
| 10222202 | 17 | 31 |
| 10222211 | 29 | 51 |

p=3 d=8

| P(T) | H(p) | h(−p) |
|---|---|---|
| 100000102 | 88 | 48 |
| 100000202 | 24 | 176 |
| 100001002 | 26 | 76 |
| 100001101 | 74 | 44 |
| 100001221 | 134 | 20 |
| 100002002 | 26 | 76 |
| 100002101 | 74 | 44 |
| 100002211 | 134 | 20 |
| 100010002 | 84 | 56 |
| 100010212 | 42 | 76 |
| 100010222 | 42 | 76 |
| 100011001 | 78 | 36 |
| 100011022 | 52 | 24 |
| 100011122 | 22 | 148 |
| 100011212 | 38 | 84 |
| 100011221 | 28 | 72 |
| 100012001 | 78 | 36 |
| 100012012 | 52 | 24 |
| 100012112 | 22 | 148 |
| 100012211 | 28 | 72 |
| 100012222 | 38 | 84 |
| 100020002 | 28 | 168 |
| 100020112 | 28 | 84 |
| 100020122 | 38 | 84 |
| 100021022 | 14 | 100 |
| 100021201 | 68 | 24 |
| 100022012 | 14 | 100 |
| 100022201 | 68 | 24 |
| 100100002 | 38 | 52 |
| 100100021 | 40 | 80 |
| 100100111 | 52 | 24 |
| 100100122 | 80 | 32 |
| 100101011 | 28 | 72 |
| 100101112 | 62 | 36 |
| 100101121 | 64 | 32 |
| 100102022 | 50 | 92 |
| 100102102 | 52 | 24 |
| 100102111 | 86 | 52 |
| 100110001 | 78 | 36 |
| 100110011 | 86 | 52 |
| 100110202 | 42 | 76 |
| 100110212 | 20 | 56 |
| 100111012 | 64 | 32 |
| 100111222 | 26 | 76 |
| 100112021 | 66 | 28 |
| 100112101 | 120 | 16 |
| 100112102 | 18 | 156 |
| 100112201 | 36 | 56 |
| 100112222 | 36 | 56 |
| 100120102 | 44 | 72 |

p=3 d=8

| P(T) | H(p) | h(−p) | P(T) | H(P) | h(−p) |
|---|---|---|---|---|---|
| 100120112 | 28 | 40 | 101010122 | 30 | 68 |
| 100120201 | 62 | 36 | 101010202 | 60 | 40 |
| 100121011 | 106 | 12 | 101011022 | 16 | 160 |
| 100121012 | 14 | 132 | 101011121 | 28 | 40 |
| 100121102 | 44 | 72 | 101012012 | 16 | 160 |
| 100121111 | 38 | 84 | 101012111 | 28 | 40 |
| 100121212 | 88 | 48 | 101020202 | 20 | 120 |
| 100122001 | 132 | 24 | 101021002 | 38 | 84 |
| 100122101 | 34 | 60 | 101021012 | 40 | 80 |
| 100200002 | 38 | 52 | 101021111 | 84 | 56 |
| 100200011 | 40 | 80 | 101021122 | 66 | 28 |
| 100200012 | 80 | 32 | 101021222 | 18 | 124 |
| 100200121 | 52 | 24 | 101022002 | 38 | 84 |
| 100201012 | 50 | 92 | 101022022 | 40 | 80 |
| 100201102 | 52 | 24 | 101022112 | 66 | 28 |
| 100201121 | 86 | 52 | 101022121 | 84 | 56 |
| 100202021 | 28 | 72 | 101022212 | 18 | 124 |
| 100202111 | 64 | 32 | 101100001 | 74 | 44 |
| 100202122 | 62 | 36 | 101100011 | 50 | 28 |
| 100210001 | 78 | 36 | 101100022 | 76 | 40 |
| 100210021 | 86 | 52 | 101100122 | 16 | 128 |
| 100210202 | 42 | 76 | 101100212 | 46 | 100 |
| 100210222 | 20 | 56 | 101101012 | 60 | 40 |
| 100211011 | 66 | 28 | 101101211 | 40 | 80 |
| 100211101 | 120 | 16 | 101101222 | 38 | 52 |
| 100211102 | 18 | 156 | 101102021 | 64 | 32 |
| 100211201 | 36 | 56 | 101102102 | 16 | 160 |
| 100211212 | 36 | 56 | 101110121 | 42 | 76 |
| 100212022 | 64 | 32 | 101110211 | 66 | 28 |
| 100212212 | 26 | 76 | 101111011 | 134 | 20 |
| 100220102 | 44 | 72 | 101111102 | 30 | 68 |
| 100220122 | 28 | 40 | 101111212 | 74 | 44 |
| 100220201 | 62 | 36 | 101112001 | 120 | 16 |
| 100221001 | 132 | 24 | 101112112 | 40 | 80 |
| 100221101 | 34 | 60 | 101112122 | 20 | 56 |
| 100222021 | 106 | 12 | 101112202 | 66 | 28 |
| 100222022 | 14 | 132 | 101112221 | 80 | 32 |
| 100222102 | 44 | 72 | 101120221 | 136 | 16 |
| 100222121 | 38 | 84 | 101120222 | 26 | 172 |
| 100222222 | 88 | 48 | 101121011 | 36 | 88 |
| 101000002 | 88 | 48 | 101121101 | 88 | 48 |
| 101000011 | 80 | 32 | 101121121 | 80 | 32 |
| 101000021 | 80 | 32 | 101122001 | 34 | 60 |
| 101000212 | 40 | 80 | 101122022 | 28 | 72 |
| 101000222 | 40 | 80 | 101122121 | 78 | 36 |
| 101001121 | 120 | 16 | 101122201 | 106 | 12 |
| 101001221 | 30 | 68 | 101122202 | 14 | 132 |
| 101002111 | 120 | 16 | 101200001 | 74 | 44 |
| 101002211 | 30 | 68 | 101200012 | 76 | 40 |
| 101010112 | 30 | 68 | 101200021 | 50 | 28 |

p=3 d=8

| P(T) | H(p) | h(−p) | P(T) | H(P) | h(−p) |
|---|---|---|---|---|---|
| 101200112 | 16 | 128 | 102012112 | 64 | 32 |
| 101200222 | 46 | 100 | 102012211 | 106 | 12 |
| 101201011 | 64 | 32 | 102012212 | 16 | 128 |
| 101201102 | 16 | 160 | 102020011 | 68 | 24 |
| 101202022 | 60 | 40 | 102020021 | 68 | 24 |
| 101202212 | 38 | 52 | 102020102 | 20 | 120 |
| 101202221 | 40 | 80 | 102020212 | 28 | 72 |
| 101210111 | 42 | 76 | 102020222 | 28 | 72 |
| 101210221 | 66 | 28 | 102021001 | 62 | 36 |
| 101211001 | 120 | 16 | 102021011 | 64 | 32 |
| 101211112 | 20 | 56 | 102021221 | 46 | 100 |
| 101211122 | 40 | 80 | 102022001 | 62 | 36 |
| 101211202 | 66 | 28 | 102022021 | 64 | 32 |
| 101211211 | 80 | 32 | 102022211 | 46 | 100 |
| 101212021 | 134 | 20 | 102100022 | 20 | 152 |
| 101212102 | 30 | 68 | 102100102 | 40 | 80 |
| 101212222 | 74 | 44 | 102100112 | 50 | 92 |
| 101220211 | 136 | 16 | 102100201 | 66 | 28 |
| 101220212 | 26 | 172 | 102100211 | 76 | 40 |
| 101221001 | 34 | 60 | 102101012 | 12 | 136 |
| 101221012 | 28 | 72 | 102101201 | 60 | 40 |
| 101221111 | 78 | 36 | 102101212 | 50 | 28 |
| 101221201 | 106 | 12 | 102102002 | 12 | 104 |
| 101221202 | 14 | 132 | 102102202 | 80 | 32 |
| 101222021 | 36 | 88 | 102102211 | 80 | 32 |
| 101222101 | 88 | 48 | 102110111 | 64 | 32 |
| 101222111 | 80 | 32 | 102110122 | 86 | 52 |
| 102000002 | 14 | 176 | 102110221 | 134 | 20 |
| 102000112 | 40 | 80 | 102111002 | 36 | 88 |
| 102000122 | 40 | 80 | 102111211 | 120 | 16 |
| 102001021 | 134 | 20 | 102111212 | 18 | 156 |
| 102001102 | 40 | 80 | 102112001 | 36 | 56 |
| 102001121 | 36 | 56 | 102120001 | 68 | 24 |
| 102001201 | 66 | 28 | 102120011 | 78 | 36 |
| 102001222 | 78 | 36 | 102120121 | 132 | 24 |
| 102002011 | 134 | 20 | 102121021 | 50 | 28 |
| 102002102 | 40 | 80 | 102121202 | 34 | 60 |
| 102002111 | 36 | 56 | 102121211 | 30 | 68 |
| 102002201 | 66 | 28 | 102122002 | 78 | 36 |
| 102002212 | 78 | 36 | 102122011 | 76 | 40 |
| 102010102 | 60 | 40 | 102122101 | 106 | 12 |
| 102011002 | 36 | 88 | 102122212 | 42 | 76 |
| 102011012 | 44 | 72 | | | |
| 102011021 | 34 | 60 | | | |
| 102011122 | 64 | 32 | | | |
| 102011221 | 106 | 12 | | | |
| 102011222 | 16 | 128 | | | |
| 102012002 | 36 | 88 | | | |
| 102012011 | 34 | 60 | | | |
| 102012022 | 44 | 72 | | | |

p=3 d=8

| P(T) | H(p) | h(−p) |
|---|---|---|
| 102200012 | 20 | 152 |
| 102200102 | 40 | 80 |
| 102200122 | 50 | 92 |
| 102200201 | 66 | 28 |
| 102200221 | 76 | 40 |
| 102201002 | 12 | 104 |
| 102201202 | 80 | 32 |
| 102201221 | 80 | 32 |
| 102202022 | 12 | 136 |
| 102202201 | 60 | 40 |
| 102202222 | 50 | 28 |
| 102210112 | 86 | 52 |
| 102210121 | 64 | 32 |
| 102210211 | 134 | 20 |
| 102211001 | 36 | 56 |
| 102212002 | 36 | 88 |
| 102212221 | 120 | 16 |
| 102212222 | 18 | 156 |
| 102220001 | 68 | 24 |
| 102220021 | 78 | 36 |
| 102220111 | 132 | 24 |
| 102221002 | 78 | 36 |
| 102221021 | 76 | 40 |
| 102221101 | 106 | 12 |
| 102221222 | 42 | 76 |
| 102222011 | 50 | 28 |
| 102222202 | 34 | 60 |
| 102222221 | 30 | 68 |

p=5 d=3

| P(T) | h(p) | h(−p) |
|---|---|---|
| 1011 | 9 | 3 |
| 1014 | 9 | 3 |
| 1021 | 7 | 5 |
| 1024 | 7 | 5 |
| 1032 | 5 | 7 |
| 1033 | 5 | 7 |
| 1042 | 3 | 9 |
| 1043 | 3 | 9 |

p=5 d=4

| P(T) | H(p) | h(−p) |
|---|---|---|
| 10002 | 2 | 20 |
| 10003 | 10 | 4 |
| 10014 | 8 | 8 |
| 10024 | 8 | 8 |
| 10034 | 8 | 8 |
| 10044 | 8 | 8 |
| 10102 | 6 | 12 |
| 10111 | 8 | 8 |
| 10122 | 6 | 12 |
| 10123 | 4 | 16 |
| 10132 | 6 | 12 |
| 10133 | 4 | 16 |
| 10141 | 8 | 8 |
| 10203 | 6 | 12 |
| 10221 | 10 | 4 |
| 10223 | 6 | 12 |
| 10231 | 10 | 4 |
| 10233 | 6 | 12 |
| 10303 | 6 | 12 |
| 10311 | 10 | 4 |
| 10313 | 6 | 12 |
| 10341 | 10 | 4 |
| 10343 | 6 | 12 |
| 10402 | 6 | 12 |
| 10412 | 6 | 12 |
| 10413 | 4 | 16 |
| 10421 | 8 | 8 |
| 10431 | 8 | 8 |
| 10442 | 6 | 12 |
| 10443 | 4 | 16 |

p=5 d=5

| | P(T) | h(p) | h(−p) |
|---|---|---|---|
| * | 100041 | 71 | 11 |
| * | 100042 | 11 | 71 |
| * | 100043 | 11 | 71 |
| * | 100044 | 71 | 11 |
| | 100102 | 25 | 37 |
| | 100201 | 37 | 25 |
| | 100304 | 37 | 25 |
| | 100403 | 25 | 37 |
| | 101022 | 23 | 35 |
| | 101023 | 23 | 35 |
| | 101032 | 15 | 51 |
| | 101033 | 15 | 51 |
| | 102001 | 37 | 25 |
| | 102004 | 37 | 25 |
| | 102012 | 19 | 55 |
| | 102013 | 19 | 55 |
| | 102021 | 51 | 15 |
| | 102024 | 51 | 15 |
| | 103002 | 25 | 37 |
| | 103003 | 25 | 37 |
| | 103011 | 55 | 19 |
| | 103014 | 55 | 19 |
| | 103022 | 15 | 51 |
| | 103023 | 15 | 51 |
| | 104021 | 35 | 23 |
| | 104024 | 35 | 23 |
| | 104031 | 51 | 15 |
| | 104034 | 51 | 15 |
| | 110004 | 35 | 23 |
| | 110014 | 35 | 23 |
| | 110041 | 15 | 51 |
| | 110123 | 17 | 53 |
| | 110131 | 33 | 21 |
| | 110142 | 29 | 41 |
| | 110144 | 69 | 9 |
| | 110202 | 35 | 23 |
| | 110213 | 27 | 39 |
| | 110232 | 31 | 19 |
| | 110243 | 23 | 35 |
| | 110244 | 13 | 49 |
| | 110301 | 55 | 19 |
| | 110303 | 15 | 27 |
| | 110322 | 31 | 19 |
| | 110331 | 49 | 13 |
| | 110333 | 29 | 41 |
| | 110343 | 27 | 39 |
| | 110403 | 15 | 51 |
| | 110411 | 39 | 27 |
| | 110421 | 33 | 21 |
| | 110432 | 17 | 29 |
| | 110441 | 31 | 19 |
| | 110442 | 21 | 33 |
| | 110444 | 71 | 11 |

p=5 d=5

| P(T) | h(p) | h(−p) |
|---|---|---|
| 120003 | 23 | 35 |
| 120013 | 23 | 35 |
| 120042 | 51 | 15 |
| 120104 | 23 | 35 |
| 120111 | 39 | 27 |
| 120134 | 19 | 31 |
| 120141 | 35 | 23 |
| 120143 | 49 | 13 |
| 120201 | 51 | 15 |
| 120212 | 27 | 39 |
| 120222 | 21 | 33 |
| 120234 | 29 | 17 |
| 120242 | 19 | 31 |
| 120243 | 11 | 71 |
| 120244 | 33 | 21 |
| 120321 | 53 | 17 |
| 120332 | 21 | 33 |
| 120343 | 9 | 69 |
| 120344 | 41 | 29 |
| 120401 | 27 | 15 |
| 120402 | 19 | 55 |
| 120424 | 19 | 31 |
| 120431 | 41 | 29 |
| 120432 | 13 | 49 |
| 120441 | 39 | 27 |

| P(T) | h(p) | h(−p) |
|---|---|---|
| 130002 | 23 | 35 |
| 130012 | 23 | 35 |
| 130043 | 51 | 15 |
| 130103 | 19 | 55 |
| 130104 | 27 | 15 |
| 130121 | 19 | 31 |
| 130133 | 13 | 49 |
| 130134 | 41 | 29 |
| 130144 | 39 | 27 |
| 130224 | 53 | 17 |
| 130233 | 21 | 33 |
| 130241 | 41 | 29 |
| 130242 | 9 | 69 |
| 130304 | 51 | 15 |
| 130313 | 27 | 39 |
| 130323 | 21 | 33 |
| 130331 | 29 | 17 |
| 130341 | 33 | 21 |
| 130342 | 11 | 71 |
| 130343 | 19 | 31 |
| 130401 | 23 | 35 |
| 130414 | 39 | 27 |
| 130431 | 19 | 31 |
| 130442 | 49 | 13 |
| 130444 | 35 | 23 |
| 140001 | 35 | 23 |
| 140011 | 35 | 23 |
| 140044 | 15 | 51 |
| 140102 | 15 | 51 |
| 140114 | 39 | 27 |
| 140124 | 33 | 21 |
| 140133 | 17 | 29 |
| 140141 | 71 | 11 |
| 140143 | 21 | 33 |
| 140144 | 31 | 19 |
| 140202 | 15 | 27 |
| 140204 | 55 | 19 |
| 140223 | 31 | 19 |
| 140232 | 29 | 41 |
| 140234 | 49 | 13 |
| 140242 | 27 | 39 |
| 140303 | 35 | 23 |
| 140312 | 27 | 39 |
| 140333 | 31 | 19 |
| 140341 | 13 | 49 |
| 140342 | 23 | 35 |
| 140422 | 17 | 53 |
| 140434 | 33 | 21 |
| 140441 | 69 | 9 |
| 140443 | 29 | 41 |

p=7 d=3

| P(T) | h(p) | h(−p) |
|---|---|---|
| 1002 | 9 | 7 |
| 1003 | 13 | 3 |
| 1004 | 3 | 13 |
| 1005 | 7 | 9 |
| 1011 | 5 | 11 |
| 1016 | 11 | 5 |
| 1021 | 5 | 11 |
| 1026 | 11 | 5 |
| 1032 | 9 | 7 |
| 1035 | 7 | 9 |
| 1041 | 5 | 11 |
| 1046 | 11 | 5 |
| 1052 | 9 | 7 |
| 1055 | 7 | 9 |
| 1062 | 9 | 7 |
| 1065 | 7 | 9 |

p=7 d=4

| P(T) | H(p) | h(−p) |
|---|---|---|
| 10011 | 12 | 8 |
| 10012 | 12 | 8 |
| 10014 | 12 | 8 |
| 10023 | 10 | 12 |
| 10025 | 10 | 12 |
| 10026 | 10 | 12 |
| 10053 | 10 | 12 |
| 10055 | 10 | 12 |
| 10056 | 10 | 12 |
| 10061 | 12 | 8 |
| 10062 | 12 | 8 |
| 10064 | 12 | 8 |
| 10103 | 10 | 12 |
| 10106 | 6 | 20 |
| 10111 | 12 | 8 |
| 10112 | 6 | 20 |
| 10121 | 10 | 12 |
| 10135 | 6 | 20 |
| 10145 | 6 | 20 |
| 10151 | 10 | 12 |
| 10161 | 12 | 8 |
| 10162 | 6 | 20 |
| 10203 | 6 | 20 |
| 10205 | 10 | 12 |
| 10211 | 6 | 20 |
| 10214 | 12 | 8 |
| 10224 | 10 | 12 |
| 10236 | 6 | 20 |
| 10246 | 6 | 20 |
| 10254 | 10 | 12 |

p=7 d=4

| P(T) | H(p) | h(−p) |
|---|---|---|
| 10261 | 6 | 20 |
| 10264 | 12 | 8 |
| 10305 | 10 | 12 |
| 10306 | 6 | 20 |
| 10316 | 10 | 12 |
| 10322 | 10 | 12 |
| 10326 | 8 | 16 |
| 10333 | 12 | 8 |
| 10334 | 8 | 16 |
| 10335 | 4 | 24 |
| 10343 | 12 | 8 |
| 10344 | 8 | 16 |
| 10345 | 4 | 24 |
| 10352 | 10 | 12 |
| 10356 | 8 | 16 |
| 10366 | 10 | 12 |
| 10405 | 6 | 20 |
| 10406 | 10 | 12 |
| 10412 | 12 | 8 |
| 10414 | 6 | 20 |
| 10422 | 10 | 12 |
| 10433 | 6 | 20 |
| 10443 | 6 | 20 |
| 10452 | 10 | 12 |
| 10462 | 12 | 8 |
| 10464 | 6 | 20 |
| 10503 | 10 | 12 |
| 10505 | 6 | 20 |
| 10515 | 10 | 12 |
| 10524 | 10 | 12 |
| 10525 | 8 | 16 |
| 10531 | 8 | 16 |
| 10533 | 4 | 24 |
| 10536 | 12 | 8 |
| 10541 | 8 | 16 |
| 10543 | 4 | 24 |
| 10546 | 12 | 8 |
| 10554 | 10 | 12 |
| 10555 | 8 | 16 |
| 10565 | 10 | 12 |
| 10603 | 6 | 20 |
| 10606 | 10 | 12 |
| 10613 | 10 | 12 |
| 10621 | 10 | 12 |
| 10623 | 8 | 16 |
| 10632 | 8 | 16 |
| 10635 | 12 | 8 |
| 10636 | 4 | 24 |
| 10642 | 8 | 16 |
| 10645 | 12 | 8 |
| 10646 | 4 | 24 |
| 10651 | 10 | 12 |
| 10653 | 8 | 16 |
| 10663 | 10 | 12 |

p=11 d=3

| P(T) | | | | h(p) | h(2p) |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 4 | 9 | 15 |
| 1 | 0 | 1 | 5 | 11 | 13 |
| 1 | 0 | 1 | 6 | 13 | 11 |
| 1 | 0 | 1 | 7 | 15 | 9 |
| 1 | 0 | 2 | 2 | 9 | 15 |
| 1 | 0 | 2 | 4 | 17 | 7 |
| 1 | 0 | 2 | 7 | 7 | 17 |
| 1 | 0 | 2 | 9 | 15 | 9 |
| 1 | 0 | 3 | 2 | 13 | 11 |
| 1 | 0 | 3 | 5 | 9 | 15 |
| 1 | 0 | 3 | 6 | 15 | 9 |
| 1 | 0 | 3 | 9 | 11 | 13 |
| 1 | 0 | 4 | 1 | 9 | 15 |
| 1 | 0 | 4 | 4 | 11 | 13 |
| 1 | 0 | 4 | 7 | 13 | 11 |
| 1 | 0 | 4 | 10 | 15 | 9 |
| 1 | 0 | 5 | 1 | 11 | 13 |
| 1 | 0 | 5 | 3 | 9 | 15 |
| 1 | 0 | 5 | 8 | 15 | 9 |
| 1 | 0 | 5 | 10 | 13 | 11 |
| 1 | 0 | 6 | 3 | 15 | 9 |
| 1 | 0 | 6 | 5 | 17 | 7 |
| 1 | 0 | 6 | 6 | 7 | 17 |
| 1 | 0 | 6 | 8 | 9 | 15 |
| 1 | 0 | 7 | 1 | 15 | 9 |
| 1 | 0 | 7 | 2 | 7 | 17 |
| 1 | 0 | 7 | 9 | 17 | 7 |
| 1 | 0 | 7 | 10 | 9 | 15 |
| 1 | 0 | 8 | 1 | 17 | 7 |
| 1 | 0 | 8 | 5 | 15 | 9 |
| 1 | 0 | 8 | 6 | 9 | 15 |
| 1 | 0 | 8 | 10 | 7 | 17 |
| 1 | 0 | 9 | 2 | 15 | 9 |
| 1 | 0 | 9 | 3 | 11 | 13 |
| 1 | 0 | 9 | 8 | 13 | 11 |
| 1 | 0 | 9 | 9 | 9 | 15 |
| 1 | 0 | 10 | 3 | 17 | 7 |
| 1 | 0 | 10 | 4 | 15 | 9 |
| 1 | 0 | 10 | 7 | 9 | 15 |
| 1 | 0 | 10 | 8 | 7 | 17 |

p=11 d=4

| P(T) | | | | | H(p) | h(2p) |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 2 | 18 | 12 |
| 1 | 0 | 0 | 1 | 3 | 10 | 28 |
| 1 | 0 | 0 | 1 | 5 | 16 | 16 |
| 1 | 0 | 0 | 2 | 4 | 16 | 16 |
| 1 | 0 | 0 | 2 | 6 | 18 | 12 |
| 1 | 0 | 0 | 2 | 9 | 10 | 28 |
| 1 | 0 | 0 | 3 | 3 | 16 | 16 |
| 1 | 0 | 0 | 3 | 4 | 10 | 28 |
| 1 | 0 | 0 | 3 | 10 | 18 | 12 |
| 1 | 0 | 0 | 4 | 1 | 16 | 16 |
| 1 | 0 | 0 | 4 | 5 | 10 | 28 |
| 1 | 0 | 0 | 4 | 7 | 18 | 12 |
| 1 | 0 | 0 | 5 | 1 | 10 | 28 |
| 1 | 0 | 0 | 5 | 8 | 18 | 12 |
| 1 | 0 | 0 | 5 | 9 | 16 | 16 |
| 1 | 0 | 0 | 6 | 1 | 10 | 28 |
| 1 | 0 | 0 | 6 | 8 | 18 | 12 |
| 1 | 0 | 0 | 6 | 9 | 16 | 16 |
| 1 | 0 | 0 | 7 | 1 | 16 | 16 |
| 1 | 0 | 0 | 7 | 5 | 10 | 28 |
| 1 | 0 | 0 | 7 | 7 | 18 | 12 |
| 1 | 0 | 0 | 8 | 3 | 16 | 16 |
| 1 | 0 | 0 | 8 | 4 | 10 | 28 |
| 1 | 0 | 0 | 8 | 10 | 18 | 12 |
| 1 | 0 | 0 | 9 | 4 | 16 | 16 |
| 1 | 0 | 0 | 9 | 6 | 18 | 12 |
| 1 | 0 | 0 | 9 | 9 | 10 | 28 |
| 1 | 0 | 0 | 10 | 2 | 18 | 12 |
| 1 | 0 | 0 | 10 | 3 | 10 | 28 |
| 1 | 0 | 0 | 10 | 5 | 16 | 16 |
| 1 | 0 | 1 | 0 | 6 | 10 | 28 |
| 1 | 0 | 1 | 0 | 7 | 18 | 12 |
| 1 | 0 | 1 | 0 | 8 | 10 | 28 |
| 1 | 0 | 1 | 1 | 2 | 14 | 20 |
| 1 | 0 | 1 | 2 | 1 | 18 | 12 |
| 1 | 0 | 1 | 2 | 2 | 8 | 32 |
| 1 | 0 | 1 | 2 | 5 | 14 | 20 |
| 1 | 0 | 1 | 2 | 8 | 8 | 32 |
| 1 | 0 | 1 | 2 | 10 | 12 | 24 |
| 1 | 0 | 1 | 3 | 9 | 14 | 20 |
| 1 | 0 | 1 | 3 | 10 | 12 | 24 |
| 1 | 0 | 1 | 4 | 3 | 16 | 16 |
| 1 | 0 | 1 | 4 | 4 | 10 | 28 |
| 1 | 0 | 1 | 4 | 7 | 6 | 36 |
| 1 | 0 | 1 | 5 | 6 | 18 | 12 |
| 1 | 0 | 1 | 5 | 8 | 14 | 20 |
| 1 | 0 | 1 | 5 | 9 | 10 | 28 |
| 1 | 0 | 1 | 5 | 10 | 12 | 24 |
| 1 | 0 | 1 | 6 | 6 | 18 | 12 |
| 1 | 0 | 1 | 6 | 8 | 14 | 20 |

p=11 d=4

| P(T) | | | | | H(p) | h(2p) |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 6 | 9 | 10 | 28 |
| 1 | 0 | 1 | 6 | 10 | 12 | 24 |
| 1 | 0 | 1 | 7 | 3 | 16 | 16 |
| 1 | 0 | 1 | 7 | 4 | 10 | 28 |
| 1 | 0 | 1 | 7 | 7 | 6 | 36 |
| 1 | 0 | 1 | 8 | 9 | 14 | 20 |
| 1 | 0 | 1 | 8 | 10 | 12 | 24 |
| 1 | 0 | 1 | 9 | 1 | 18 | 12 |
| 1 | 0 | 1 | 9 | 2 | 8 | 32 |
| 1 | 0 | 1 | 9 | 5 | 14 | 20 |
| 1 | 0 | 1 | 9 | 8 | 8 | 32 |
| 1 | 0 | 1 | 9 | 10 | 12 | 24 |
| 1 | 0 | 1 | 10 | 2 | 14 | 20 |
| 1 | 0 | 2 | 0 | 2 | 14 | 20 |
| 1 | 0 | 2 | 0 | 6 | 6 | 36 |
| 1 | 0 | 2 | 0 | 10 | 14 | 20 |
| 1 | 0 | 2 | 1 | 4 | 10 | 28 |
| 1 | 0 | 2 | 1 | 6 | 14 | 20 |
| 1 | 0 | 2 | 2 | 3 | 12 | 24 |
| 1 | 0 | 2 | 2 | 7 | 12 | 24 |
| 1 | 0 | 2 | 2 | 9 | 18 | 12 |
| 1 | 0 | 2 | 3 | 1 | 16 | 16 |
| 1 | 0 | 2 | 3 | 6 | 12 | 24 |
| 1 | 0 | 2 | 3 | 7 | 12 | 24 |
| 1 | 0 | 2 | 4 | 7 | 12 | 24 |
| 1 | 0 | 2 | 5 | 1 | 14 | 20 |
| 1 | 0 | 2 | 5 | 5 | 16 | 16 |
| 1 | 0 | 2 | 5 | 9 | 12 | 24 |
| 1 | 0 | 2 | 6 | 1 | 14 | 20 |
| 1 | 0 | 2 | 6 | 5 | 16 | 16 |
| 1 | 0 | 2 | 6 | 9 | 12 | 24 |
| 1 | 0 | 2 | 7 | 7 | 12 | 24 |
| 1 | 0 | 2 | 8 | 1 | 16 | 16 |
| 1 | 0 | 2 | 8 | 6 | 12 | 24 |
| 1 | 0 | 2 | 8 | 7 | 12 | 24 |
| 1 | 0 | 2 | 9 | 3 | 12 | 24 |
| 1 | 0 | 2 | 9 | 7 | 12 | 24 |
| 1 | 0 | 2 | 9 | 9 | 18 | 12 |
| 1 | 0 | 2 | 10 | 4 | 10 | 28 |
| 1 | 0 | 2 | 10 | 6 | 14 | 20 |
| 1 | 0 | 3 | 0 | 6 | 10 | 28 |
| 1 | 0 | 3 | 0 | 8 | 18 | 12 |
| 1 | 0 | 3 | 0 | 10 | 10 | 28 |
| 1 | 0 | 3 | 1 | 2 | 12 | 24 |
| 1 | 0 | 3 | 1 | 4 | 14 | 20 |
| 1 | 0 | 3 | 2 | 2 | 12 | 24 |
| 1 | 0 | 3 | 2 | 4 | 10 | 28 |
| 1 | 0 | 3 | 2 | 6 | 14 | 20 |
| 1 | 0 | 3 | 2 | 10 | 18 | 12 |
| 1 | 0 | 3 | 3 | 1 | 14 | 20 |
| 1 | 0 | 3 | 3 | 2 | 12 | 24 |
| 1 | 0 | 3 | 3 | 6 | 8 | 32 |
| 1 | 0 | 3 | 3 | 7 | 8 | 32 |
| 1 | 0 | 3 | 3 | 9 | 18 | 12 |
| 1 | 0 | 3 | 4 | 7 | 14 | 20 |

| P(T) | | | | | H(p) | h(2p) |
|---|---|---|---|---|---|---|
| 1 | 0 | 3 | 5 | 3 | 10 | 28 |
| 1 | 0 | 3 | 5 | 5 | 16 | 16 |
| 1 | 0 | 3 | 5 | 8 | 6 | 36 |
| 1 | 0 | 3 | 6 | 3 | 10 | 28 |
| 1 | 0 | 3 | 6 | 5 | 16 | 16 |
| 1 | 0 | 3 | 6 | 8 | 6 | 36 |
| 1 | 0 | 3 | 7 | 7 | 14 | 20 |
| 1 | 0 | 3 | 8 | 1 | 14 | 20 |
| 1 | 0 | 3 | 8 | 2 | 12 | 24 |
| 1 | 0 | 3 | 8 | 6 | 8 | 32 |
| 1 | 0 | 3 | 8 | 7 | 8 | 32 |
| 1 | 0 | 3 | 8 | 9 | 18 | 12 |
| 1 | 0 | 3 | 9 | 2 | 12 | 24 |
| 1 | 0 | 3 | 9 | 4 | 10 | 28 |
| 1 | 0 | 3 | 9 | 6 | 14 | 20 |
| 1 | 0 | 3 | 9 | 10 | 18 | 12 |
| 1 | 0 | 3 | 10 | 2 | 12 | 24 |
| 1 | 0 | 3 | 10 | 4 | 14 | 20 |
| 1 | 0 | 4 | 0 | 2 | 18 | 12 |
| 1 | 0 | 4 | 0 | 7 | 10 | 28 |
| 1 | 0 | 4 | 0 | 8 | 10 | 28 |
| 1 | 0 | 4 | 1 | 2 | 6 | 36 |
| 1 | 0 | 4 | 1 | 4 | 16 | 16 |
| 1 | 0 | 4 | 1 | 9 | 10 | 28 |
| 1 | 0 | 4 | 2 | 1 | 14 | 20 |
| 1 | 0 | 4 | 2 | 6 | 12 | 24 |
| 1 | 0 | 4 | 3 | 10 | 14 | 20 |
| 1 | 0 | 4 | 4 | 1 | 10 | 28 |
| 1 | 0 | 4 | 4 | 6 | 12 | 24 |
| 1 | 0 | 4 | 4 | 7 | 14 | 20 |
| 1 | 0 | 4 | 4 | 8 | 18 | 12 |
| 1 | 0 | 4 | 5 | 3 | 14 | 20 |
| 1 | 0 | 4 | 5 | 5 | 18 | 12 |
| 1 | 0 | 4 | 5 | 6 | 12 | 24 |
| 1 | 0 | 4 | 5 | 7 | 8 | 32 |
| 1 | 0 | 4 | 5 | 10 | 8 | 32 |
| 1 | 0 | 4 | 6 | 3 | 14 | 20 |
| 1 | 0 | 4 | 6 | 5 | 18 | 12 |
| 1 | 0 | 4 | 6 | 6 | 12 | 24 |
| 1 | 0 | 4 | 6 | 7 | 8 | 32 |
| 1 | 0 | 4 | 6 | 10 | 8 | 32 |
| 1 | 0 | 4 | 7 | 1 | 10 | 28 |
| 1 | 0 | 4 | 7 | 6 | 12 | 24 |
| 1 | 0 | 4 | 7 | 7 | 14 | 20 |
| 1 | 0 | 4 | 7 | 8 | 18 | 12 |
| 1 | 0 | 4 | 8 | 10 | 14 | 20 |
| 1 | 0 | 4 | 9 | 1 | 14 | 20 |
| 1 | 0 | 4 | 9 | 6 | 12 | 24 |
| 1 | 0 | 4 | 10 | 2 | 6 | 36 |
| 1 | 0 | 4 | 10 | 4 | 16 | 16 |
| 1 | 0 | 4 | 10 | 9 | 10 | 28 |
| 1 | 0 | 5 | 0 | 2 | 10 | 28 |
| 1 | 0 | 5 | 0 | 7 | 10 | 28 |
| 1 | 0 | 5 | 0 | 10 | 18 | 12 |
| 1 | 0 | 5 | 1 | 2 | 14 | 20 |

p=11 d=4

| P(T) | H(P) | h(2p) | P(T) | H(p) | h(2p) |
|---|---|---|---|---|---|
| 1 0 5 1 5 | 10 | 28 | 1 0 6 10 10 | 12 | 24 |
| 1 0 5 1 7 | 18 | 12 | 1 0 7 0 2 | 6 | 36 |
| 1 0 5 1 8 | 12 | 24 | 1 0 7 0 7 | 14 | 20 |
| 1 0 5 2 6 | 14 | 20 | 1 0 7 0 8 | 14 | 20 |
| 1 0 5 3 1 | 10 | 28 | 1 0 7 1 1 | 12 | 24 |
| 1 0 5 3 9 | 16 | 16 | 1 0 7 1 3 | 18 | 12 |
| 1 0 5 3 10 | 6 | 36 | 1 0 7 1 6 | 12 | 24 |
| 1 0 5 4 2 | 8 | 32 | 1 0 7 2 6 | 12 | 24 |
| 1 0 5 4 3 | 18 | 12 | 1 0 7 3 3 | 12 | 24 |
| 1 0 5 4 4 | 14 | 20 | 1 0 7 3 4 | 14 | 20 |
| 1 0 5 4 6 | 8 | 32 | 1 0 7 3 9 | 16 | 16 |
| 1 0 5 4 8 | 12 | 24 | 1 0 7 4 2 | 12 | 24 |
| 1 0 5 5 5 | 14 | 20 | 1 0 7 4 4 | 16 | 16 |
| 1 0 5 5 8 | 12 | 24 | 1 0 7 4 6 | 12 | 24 |
| 1 0 5 6 5 | 14 | 20 | 1 0 7 5 2 | 14 | 20 |
| 1 0 5 6 8 | 12 | 24 | 1 0 7 5 5 | 10 | 28 |
| 1 0 5 7 2 | 8 | 32 | 1 0 7 6 2 | 14 | 20 |
| 1 0 5 7 3 | 18 | 12 | 1 0 7 6 5 | 10 | 28 |
| 1 0 5 7 4 | 14 | 20 | 1 0 7 7 2 | 12 | 24 |
| 1 0 5 7 6 | 8 | 32 | 1 0 7 7 4 | 16 | 16 |
| 1 0 5 7 8 | 12 | 24 | 1 0 7 7 6 | 12 | 24 |
| 1 0 5 8 1 | 10 | 28 | 1 0 7 8 3 | 12 | 24 |
| 1 0 5 8 9 | 16 | 16 | 1 0 7 8 4 | 14 | 20 |
| 1 0 5 8 10 | 6 | 36 | 1 0 7 8 9 | 16 | 16 |
| 1 0 5 9 6 | 14 | 20 | 1 0 7 9 6 | 12 | 24 |
| 1 0 5 10 2 | 14 | 20 | 1 0 7 10 1 | 12 | 24 |
| 1 0 5 10 5 | 10 | 28 | 1 0 7 10 3 | 18 | 12 |
| 1 0 5 10 7 | 18 | 12 | 1 0 7 10 6 | 12 | 24 |
| 1 0 5 10 8 | 12 | 24 | 1 0 8 0 6 | 14 | 20 |
| 1 0 6 0 2 | 14 | 20 | 1 0 8 0 8 | 6 | 36 |
| 1 0 6 0 7 | 14 | 20 | 1 0 8 0 10 | 14 | 20 |
| 1 0 6 0 10 | 6 | 36 | 1 0 8 1 2 | 12 | 24 |
| 1 0 6 1 8 | 12 | 24 | 1 0 8 2 2 | 12 | 24 |
| 1 0 6 1 9 | 16 | 16 | 1 0 8 2 5 | 16 | 16 |
| 1 0 6 1 10 | 12 | 24 | 1 0 8 2 8 | 12 | 24 |
| 1 0 6 2 1 | 16 | 16 | 1 0 8 3 8 | 14 | 20 |
| 1 0 6 2 4 | 12 | 24 | 1 0 8 3 9 | 10 | 28 |
| 1 0 6 2 9 | 14 | 20 | 1 0 8 4 1 | 12 | 24 |
| 1 0 6 3 4 | 18 | 12 | 1 0 8 4 3 | 16 | 16 |
| 1 0 6 3 5 | 12 | 24 | 1 0 8 4 5 | 14 | 20 |
| 1 0 6 3 8 | 12 | 24 | 1 0 8 5 1 | 18 | 12 |
| 1 0 6 4 3 | 10 | 28 | 1 0 8 5 2 | 12 | 24 |
| 1 0 6 4 10 | 14 | 20 | 1 0 8 5 4 | 12 | 24 |
| 1 0 6 5 8 | 12 | 24 | 1 0 8 6 1 | 18 | 12 |
| 1 0 6 6 8 | 12 | 24 | 1 0 8 6 2 | 12 | 24 |
| 1 0 6 7 3 | 10 | 28 | 1 0 8 6 4 | 12 | 24 |
| 1 0 6 7 10 | 14 | 20 | 1 0 8 7 1 | 12 | 24 |
| 1 0 6 8 4 | 18 | 12 | 1 0 8 7 3 | 16 | 16 |
| 1 0 6 8 5 | 12 | 24 | 1 0 8 7 5 | 14 | 20 |
| 1 0 6 8 8 | 12 | 24 | 1 0 8 8 8 | 14 | 20 |
| 1 0 6 9 1 | 16 | 16 | 1 0 8 8 9 | 10 | 28 |
| 1 0 6 9 4 | 12 | 24 | 1 0 8 9 2 | 12 | 24 |
| 1 0 6 9 9 | 14 | 20 | 1 0 8 9 5 | 16 | 16 |
| 1 0 6 10 8 | 12 | 24 | 1 0 8 9 8 | 12 | 24 |
| 1 0 6 10 9 | 16 | 16 | 1 0 8 10 2 | 12 | 24 |

p=11 d=4

| P(T) | | | | | H(p) | h(2p) |
|---|---|---|---|---|---|---|
| 1 | 0 | 9 | 0 | 2 | 10 | 28 |
| 1 | 0 | 9 | 0 | 6 | 18 | 12 |
| 1 | 0 | 9 | 0 | 10 | 10 | 28 |
| 1 | 0 | 9 | 1 | 4 | 18 | 12 |
| 1 | 0 | 9 | 1 | 7 | 12 | 24 |
| 1 | 0 | 9 | 1 | 8 | 8 | 32 |
| 1 | 0 | 9 | 1 | 9 | 14 | 20 |
| 1 | 0 | 9 | 1 | 10 | 8 | 32 |
| 1 | 0 | 9 | 2 | 1 | 16 | 16 |
| 1 | 0 | 9 | 2 | 5 | 10 | 28 |
| 1 | 0 | 9 | 2 | 6 | 6 | 36 |
| 1 | 0 | 9 | 3 | 2 | 18 | 12 |
| 1 | 0 | 9 | 3 | 3 | 10 | 28 |
| 1 | 0 | 9 | 3 | 7 | 12 | 24 |
| 1 | 0 | 9 | 3 | 10 | 14 | 20 |
| 1 | 0 | 9 | 4 | 3 | 14 | 20 |
| 1 | 0 | 9 | 4 | 7 | 12 | 24 |
| 1 | 0 | 9 | 5 | 8 | 14 | 20 |
| 1 | 0 | 9 | 6 | 8 | 14 | 20 |
| 1 | 0 | 9 | 7 | 3 | 14 | 20 |
| 1 | 0 | 9 | 7 | 7 | 12 | 24 |
| 1 | 0 | 9 | 8 | 2 | 18 | 12 |
| 1 | 0 | 9 | 8 | 3 | 10 | 28 |
| 1 | 0 | 9 | 8 | 7 | 12 | 24 |
| 1 | 0 | 9 | 8 | 10 | 14 | 20 |
| 1 | 0 | 9 | 9 | 1 | 16 | 16 |
| 1 | 0 | 9 | 9 | 5 | 10 | 28 |
| 1 | 0 | 9 | 9 | 6 | 6 | 36 |
| 1 | 0 | 9 | 10 | 4 | 18 | 12 |
| 1 | 0 | 9 | 10 | 7 | 12 | 24 |
| 1 | 0 | 9 | 10 | 8 | 8 | 32 |
| 1 | 0 | 9 | 10 | 9 | 14 | 20 |
| 1 | 0 | 9 | 10 | 10 | 8 | 32 |
| 1 | 0 | 10 | 0 | 6 | 14 | 20 |
| 1 | 0 | 10 | 0 | 7 | 6 | 36 |
| 1 | 0 | 10 | 0 | 8 | 14 | 20 |
| 1 | 0 | 10 | 1 | 3 | 14 | 20 |
| 1 | 0 | 10 | 1 | 4 | 16 | 16 |
| 1 | 0 | 10 | 1 | 5 | 12 | 24 |
| 1 | 0 | 10 | 2 | 1 | 10 | 28 |
| 1 | 0 | 10 | 2 | 7 | 14 | 20 |
| 1 | 0 | 10 | 3 | 10 | 12 | 24 |

| P(T) | | | | | H(P) | h(2p) |
|---|---|---|---|---|---|---|
| 1 | 0 | 10 | 4 | 5 | 18 | 12 |
| 1 | 0 | 10 | 4 | 9 | 12 | 24 |
| 1 | 0 | 10 | 4 | 10 | 12 | 24 |
| 1 | 0 | 10 | 5 | 3 | 16 | 16 |
| 1 | 0 | 10 | 5 | 7 | 12 | 24 |
| 1 | 0 | 10 | 5 | 10 | 12 | 24 |
| 1 | 0 | 10 | 6 | 3 | 16 | 16 |
| 1 | 0 | 10 | 6 | 7 | 12 | 24 |
| 1 | 0 | 10 | 6 | 10 | 12 | 24 |
| 1 | 0 | 10 | 7 | 5 | 18 | 12 |
| 1 | 0 | 10 | 7 | 9 | 12 | 24 |
| 1 | 0 | 10 | 7 | 10 | 12 | 24 |
| 1 | 0 | 10 | 8 | 10 | 12 | 24 |
| 1 | 0 | 10 | 9 | 1 | 10 | 28 |
| 1 | 0 | 10 | 9 | 7 | 14 | 20 |
| 1 | 0 | 10 | 10 | 3 | 14 | 20 |
| 1 | 0 | 10 | 10 | 4 | 16 | 16 |
| 1 | 0 | 10 | 10 | 5 | 12 | 24 |