On the Arnold conjecture for weakly monotone symplectic manifolds

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Summary: Hofer and Salamon [H-S] proved the Arnold conjecture in the case that the symplectic manifold is monotone (due to Floer), or $c_1 = 0$, or the minimal Chern number N is at least half of the dimension of M. In this note, we shall prove the conjecture in the case of weakly monotone symplectic manifolds, the notion of which was introduced by McDuff under the name of "semi-positive". Floer and Hofer [F-H] considered "relative" Floer homology groups. We define the modified Floer homology group by the direct limit of the inverse limit of relative Floer homology groups $HF_{*,(i,j)}$. To compute this group, we compare the relative complex $C_{*,(i,j)}$ with the relative Floer complex of $\epsilon(i,j) \cdot f$, which is independent of the choice of $\epsilon(i,j)$ sufficiently small. Chain homomorphisms between them are obtained as in [H-S] and compatible with natural homomorphisms among relative Floer homology groups.

1 Introduction.

A diffeomorphism ϕ on a symplectic manifold (M, ω) is called an exact symplectomorphism, if ϕ is the time 1 map of a time-dependent Hamiltonian vector field. A fixed point p is said to be non-degenerate, if 1 is not an eigenvalue of the differential $d\phi : T_pM \to T_pM$. From now on, we assume that M is compact. Arnold conjectured that the number of fixed points of an exact symplectomorphism is estimated below by the sum of the Betti numbers of M, if all the fixed points are non-degenerate. It is well-known that there is a one-to-one correspondence between fixed points of ϕ and 1-periodic solutions of a certain Hamiltonian system ([C-Z]). The periodic Hamiltonian equation is the Euler-Lagrange equation of the action functional on (a certain covering space of) the loop space of M (see §2). Floer developed an analogue of Morse theory for the action functinal, which is now called Floer homology theory.

A symplectic structure ω determines an almost complex structure unique up to homotopy and we denote by $c_1 = c_1(M)$ the first Chern class of TM. (M, ω) is called monotone, if there exists $\lambda > 0$ such that $c_1(A) = \lambda\omega(A)$ for any 2-homology class represented by a continuous mapping from the 2-sphere. Floer [F] proved he Arnold conjecture for monotone symplectic manifolds. Hofer and Salamon [H-S] refined the argument and proved the Arnold conjecture in the following cases:
(i) (M, ω) is monotone.
(ii) $c_1 = 0$.
(iii) The minimal Chern number is at least $1/2 \cdot \dim M$.
Here the minimal Chern number is the least non-negative integer among $c_1(A)$ for $A \in \operatorname{Im}\{\pi_2(M) \to H_2(M; Z)\}$. (M, ω) is called weakly monotone (or semi-positive [MD]), if $\omega(A) \leq 0$ for any $A \in \pi_2(M)$ with $3 - n \leq c_1(A) \leq 0$. Actually Hofer and Salamon defined Floer homology groups for periodic Hamiltonian systems on weakly monotone symplectic manifolds. However it is necessary for computation of Floer homology groups that all the connecting orbits of relative index less than 2 should be handled simultaneously. The weak-compactness argument requires the upper bound of the energy functional and they avoid this difficulty by assuming one of the conditions above.

In this note, we introduce a filtration on the Floer complex and modify the Floer homology group such that we have the upper bound of the energy functional for each stage, which yields the following

Theorem 1.1 *Let (M, ω) be a weakly monotone symplectic manifold, and ϕ an exact symplectomorphism. If all the fixed points of ϕ are non-degenerate, the number of fixed points of ϕ is bounded below by $\sum b_p(M; \mathbf{Z}/2)$, where $b_p(M; \mathbf{Z}/2)$ denotes the p-th Betti number of M with $\mathbf{Z}/2$-coefficient.*

If $\dim M \leq 6$, (M, ω) is automatically weakly monotone and the Arnold conjecture holds. We shall show this result by estimating the number of contractible periodic solutions of a periodic Hamiltonian system whose time 1 map is ϕ.

2 Preliminaries.

We recall known facts on Floer homology of periodic Hamiltonian systems. Details are found in [F],[H-S],[S-Z].

Let (M,ω) be a closed symplectic manifold and $H : M \times S^1 \to \mathbf{R}$ a smooth function, called a periodic Hamiltonian function. $\mathcal{P}(H)$ denotes the set of all contractible loops satisfying

$$\dot{x}(t) + X_H(t, x(t)) = 0 \tag{2.1}$$

where X_H is the Hamiltonian vector field of H. If $\pi_2(M) = 0$, the equation (2.1) is the Euler-Lagrange equation of the action functional $a_H : \mathcal{L}(M) \to \mathbf{R}$ on the space of contractible loops in M defined as follows:

$$a_H(x) = -\int_{D^2} u^*\omega + \int_0^1 H(t, x(t))dt \tag{2.2}$$

where u is the bounding disk of x, i.e. $u|_{\partial D^2} = x$. If $\pi_2(M) \neq 0$, the first term of the right-hand-side of (2.2) is not well-defined. However it is well-defined over the covering space $\tilde{\mathcal{L}}(M)$ of $\mathcal{L}(M)$ corresponding to the homomorphism $\phi_\omega : \pi_2(M) \to \mathbf{R}$. After [H-S], we introduce the space $\tilde{\mathcal{L}}(M)$ as follows:

$$\tilde{\mathcal{L}}(M) = \{(x,u) | x \in \mathcal{L}(M), u : D^2 \to M \text{ such that } x = u|_{\partial D^2}\}/\sim$$

$$(x,u) \sim (y,v) \Leftrightarrow \begin{cases} x = y \\ \int_{D^2} u^*\omega = \int_{D^2} v^*\omega \\ \int_{D^2} u^*c_1 = \int_{D^2} v^*c_1 \end{cases}$$

The covering transformation group of $\tilde{\mathcal{L}}(M) \to \mathcal{L}(M)$ is

$$\Gamma = \frac{\pi_2(M)}{\ker \phi_{c_1} \cap \ker \phi_\omega}.$$

Geometrically, $\pi_2(M)$ acts on $\tilde{\mathcal{L}}(M)$ by connected sum of 2-spheres with the bounding disk. Λ_ω denotes the completion of the group ring of Γ over a field $\mathbf{Z}/2$ with respect to the weight homomorphism $\phi_\omega : \pi_2(M) \to \mathbf{R}$, i.e. the set of all formal sums $\sum_A \lambda_A \cdot \delta_A$ satisfying that

$$\{A \in \Gamma | \lambda_A \neq 0, \phi_\omega(A) < c\} \text{ is finite for any } c \in \mathbf{R}.$$

We introduce the grading of δ_A as $2c_1(A)$. Fix an almost complex structure J calibrated by ω and consider the space $\mathcal{M}([x^-, u^-], [x^+, u^+])$ of the trajectories of the "(minus) gradient flow" of a_H from $[x^-, u^-]$ to $[x^+, u^+]$, i.e. solutions of the following:

$$\mathcal{F}u = \frac{\partial u}{\partial s} + J(u)\frac{\partial u}{\partial t} + \nabla H(t, u) = 0 \tag{2.3}$$

$$\lim_{s \to -\infty} u(s, t) = x^-(t), \lim_{s \to +\infty} u(s, t) = x^+(t) \tag{2.4}$$

$$(x^+, u^- \sharp u) \sim (x^+, u^+). \tag{2.5}$$

This equation is invariant under translations in s-variable and $\mathbf{R}$ acts on $\mathcal{M}([x^-, u^-], [x^+, u^+])$ freely unless $[x^-, u^-] = [x^+, u^+]$. The linearized operator of $\mathcal{F}$ at u is

$$F_u \xi = \nabla_s \xi + J(u)\nabla_t \xi + \nabla_\xi J(u)\frac{\partial u}{\partial t} + \nabla_\xi \nabla H(t, u). \tag{2.6}$$

$\tilde{\mathcal{P}}(H)$ denotes the inverse image of $\mathcal{P}(H)$ by the projection $\tilde{\mathcal{L}}(M) \to \mathcal{L}(M)$, then there is the Conley-Zender index $\mu : \tilde{\mathcal{P}}(H) \to \mathbf{Z}$ (see:[H-S],[S-Z]), which satisfies index $F_u = \mu([x^-, u^-]) - \mu([x^+, u^+])$ for $[x^\pm, u^\pm] \in \tilde{\mathcal{P}}(H)$. The Sard-Smale theorem [Sm] yields that $\mathcal{M}([x^-, u^-], [x^+, u^+])$ is a manifold of dimension $\mu([x^-, u^-]) - \mu([x^+, u^+])$ for a generic pair (J, H). The energy of a solution u of (2.3),(2.4), (2.5) is defined as follows:

$$E(u) = \frac{1}{2}\int_{-\infty}^{\infty}\int_0^1 (\left|\frac{\partial u}{\partial s}\right|^2 + \left|\frac{\partial u}{\partial t} + X_H(t, u)\right|^2) dt ds. \tag{2.7}$$

For $u \in \mathcal{M}(\tilde{x}, \tilde{y})$, we have

$$E(u) = a_H(\tilde{x}) - a_H(\tilde{y}).$$

An $2n$-dimensional symplectic manifold (M, ω) is called weakly monotone if it satisfies $\omega(A) \leq 0$ for any $A \in \pi_2(M)$ with $3 - n \leq c_1(A) < 0$ [H-S]. This condition yields the non-existence of J-holomorphic spheres of negative

Chern number for a generic almost complex structure J. C_k denotes the $\mathbf{Z}/2$-vector space consisting of $\sum_{\mu(\tilde{x})=k}\xi(\tilde{x})\cdot\tilde{x}$ where the coefficients $\xi(\tilde{x})$ satisfy the following finiteness condition.

$$\{\tilde{x} \mid \xi(\tilde{x}) \neq 0, \text{ and } a_H(\tilde{x}) > c\} \text{ is a finite set for any } c \in \mathbf{R}.$$

The boundary operator is defined as follows:

$$\partial\tilde{x} = \sum_{\mu(\tilde{y})=\mu(\tilde{x})-1} n_2(\tilde{x},\tilde{y})\cdot\tilde{y},$$

where $n_2(\tilde{x},\tilde{y})$ is the modulo 2-reduction of the cardinality of $\mathcal{M}(\tilde{x},\tilde{y})/\mathbf{R}$. (C_*,∂) is called the Floer chain complex associated to (H,J). Hofer and Salamon showed $\partial^2 = 0$ for weakly monotone symplectic manifolds [H-S,Theorem 5.1]. $C_* = \oplus_k C_k$ is a graded module over a graded algebra Λ_ω and ∂ is Λ_ω-linear. Hence the homology group $HF_*(H,J)$ of (C_*,∂) is a graded Λ_ω-module. Moreover they proved the following

Theorem 2.8 ([H-S,Theorem 5.2]) *For generic pairs $(H^\alpha,J^\alpha),(H^\beta,J^\beta)$, there exists a natural Λ_ω-module homomorphism*

$$HF^{\beta\alpha} : HF_*(H^\alpha,J^\beta) \to HF_*(H^\beta,J^\beta)$$

which preserves the grading by the Conley-Zehnder index. If (H^γ,J^γ) is any other such pair then

$$HF^{\gamma\beta}\circ HF^{\beta\alpha} = HF^{\gamma\alpha}, HF^{\alpha\alpha} = id.$$

In particular, $HF^{\beta\alpha}$ is a Λ_ω-module isomorphism.

For the proof of this theorem, they considered s-depending analogue of the equation (2.3). For generic pairs (H^α,J^α) and (H^β,J^β), we choose a path $\{(H_s,J_s)|s\in\mathbf{R}\}$ which satisfies

(2.9) $(H_s,J_s) = (H^\alpha,J^\alpha)$ for $s < -R$, $(H_s,J_s) = (H^\beta,J^\beta)$ for $s > +R$

for some positive real number R. Let $\tilde{z} = (z,u^-) \in \tilde{\mathcal{P}}(H^\alpha)$ and $\tilde{w} = (w,u^+) \in \tilde{\mathcal{P}}(H^\beta)$. $\mathcal{M}(\tilde{z},\tilde{w};\{H_s\})$ denotes the space of solutions of the following

$$\frac{\partial u}{\partial s} + J_s(u)\frac{\partial u}{\partial t} + \nabla H_s(t,u) = 0 \qquad (2.10)$$

$$(2.11)\qquad \lim_{s\to-\infty} u(s,t) = z(t),\ \lim_{s\to+\infty} u(s,t) = w(t)$$

$$(2.12)\qquad (w, u^-\sharp u) \sim (w, u^+).$$

We define the energy of $u : \mathbf{R}\times S^1 \to M$ satisfying the asymptotic condition (2.11) as follows:

$$(2.13)\qquad E_{\{H_s\}}(u) = \frac{1}{2}\int_{-\infty}^{+\infty}\int_0^1 \left(\left|\frac{\partial u}{\partial s}\right|^2 + \left|\frac{\partial u}{\partial t} + X_{H_s}(t,u)\right|^2\right) dt ds$$

If u is a solution of (2.10), the energy of u is finite if and only if u satisfies the asymptotic condition (2.11) for some $\tilde{z}$ and $\tilde{w}$. We also have the following estimate of the energy.

$$(2.14)\ \left|E_{\{H_s\}}(u) - \{a_{H^\alpha}(\tilde{z}) - a_{H^\beta}(\tilde{w})\}\right| \le \int_{-\infty}^{+\infty} \max_{x\in M, t\in S^1} \left|\frac{\partial}{\partial s} H_s(t,x)\right| ds$$

Remark that the condition (2.9) assures that the last term in the right hand side of (2.14) is finite. The argument for (2.3) yields that $\mathcal{M}(\tilde{z},\tilde{w};\{H_s\})$ is a manifold of dimension $\mu_{H^\alpha}(\tilde{z}) - \mu_{H^\beta}(\tilde{w})$ for a generic path $\{H_s\}$. Since we have a uniform bound of the energy, the weak compactness holds. In particular, $\mathcal{M}(\tilde{z},\tilde{w};\{H_s\})$ is a finite set if $\mu_{H^\alpha}(\tilde{z}) = \mu_{H^\beta}(\tilde{w})$. We define a Λ_ω-module homomorphism $\phi^{\beta\alpha} : C_*(H^\alpha, J^\alpha) \to C_*(H^\beta, J^\beta)$ by

$$\phi^{\beta\alpha}(\tilde{z}) = \sum_{\mu_{H^\alpha}(\tilde{z}) = \mu_{H^\beta}(\tilde{w})} m_2(\tilde{z},\tilde{w})\cdot\tilde{w},$$

where $m_2(\tilde{z},\tilde{w})$ is the modulo 2-reduction of the cardinality of $\mathcal{M}(\tilde{z},\tilde{w};\{H_s\})$. Investigation on the end of 1-dimensional components of $\mathcal{M}(\tilde{z},\tilde{w};\{H_s\})$ yields that $\phi^{\beta\alpha}$ is a Λ_ω-linear chain homomorphism. The induced homomorphism $HF^{\beta\alpha}$ between homology groups does not depend on the choice of generic paths satisfying (2.9), since we have a uniform bound of the energy, hence the weak-compactness of solutions of (2.10) once we fix a homotopy, between two given generic paths, through paths satisfying (2.9) for some fixed R.

Let $(H_s^{(1)}, J_s^{(1)})$ and $(H_s^{(2)}, J_s^{(2)})$ be paths satisfying

$$(H_s^{(1)}, J_s^{(1)}) = (H^\alpha, J^\alpha) \text{ for } s < -R,\ (H_s^{(1)}, J_s^{(1)}) = (H^\beta, J^\beta) \text{ for } s > +R$$

$$(H_s^{(2)}, J_s^{(2)}) = (H^\beta, J^\beta) \text{ for } s < -R, \ (H_s^{(2)}, J_s^{(2)}) = (H^\gamma, J^\gamma) \text{ for } s > +R$$

for some R. To show $HF^{\gamma\alpha} = HF^{\gamma\beta} \circ HF^{\beta\alpha}$, we have to consider the following family of paths.

$$H_{s,\lambda} = \begin{cases} H_{s+R+\lambda} & \text{for} \quad s < -\lambda \\ H^\beta & \text{for} \quad -\lambda \leq s \leq \lambda \\ H_{s-R-\lambda} & \text{for} \quad s > \lambda \end{cases}$$

For the above family of paths, it is easy to see that the last term of (2.14) is uniformly bounded with respect to $\lambda > 0$. The gluing argument relates $\mathcal{M}(\{H_s^{(1)}\})$, $\mathcal{M}(\{H_s^{(2)}\})$ and $\mathcal{M}(\{H_{s,\lambda}\})$ for a sufficiently large λ, which yields $HF^{\gamma\beta} \circ HF^{\beta\alpha} = HF^{\gamma\alpha}$.

Hofer and Salamon computed the Floer homology for a generic pair (H, J) under certain conditions.

Theorem 2.15 ([H-S,Theorem 6.1]) *Assume either that (M, ω) is monotone or $c_1(\pi_2(M)) = 0$ or the minimal Chern number is $N \geq n$. Then for a generic pair (H^α, J^α), there exists a natural homomorphism*

$$HF^\alpha : HF_*(H^\alpha, J^\alpha) \to H_{*+n}(M; \mathbf{Z}/2) \otimes \Lambda_\omega.$$

If (H^β, J^β) is any other such pair, then $HF^\beta \circ HF^{\beta\alpha} = HF^\alpha$.

3 Filtered Floer complex.

In this section, we assume that the symplectic form ω has integral periods, i.e. $[\omega] \in \mathrm{Im}\{H^2(M; \mathbf{Z}) \to H^2(M; \mathbf{R})\}$. For a fixed Hamiltonian H, we can choose a sequence of real numbers $\{r_j : j \in \mathbf{Z}\}$ satisfying

(i) $r_j \to \pm\infty$ as $j \to \pm\infty$,

(ii) $\{r_j\}$ does not contain critical values of a_H.

$C_{*,j} := \{\sum \xi(\tilde{x}) \cdot \tilde{x} \in C_* \mid \xi(\tilde{x}) = 0 \text{ if } a_H(\tilde{x}) > r_j\}$ is a subcomplex of the Floer complex C_*. We define the "relative homology" of the pair $(C_{*,j}, C_{*,i})$ $(i < j)$, i.e. the homology group of $C_{*,(i,j)} = C_{*,j}/C_{*,i}$:

$$HF_{*,(i,j)} = H_*(C_{*,j}/C_{*,i}, \partial).$$

We have the following commutative diagram:

$$
\begin{array}{ccccccccc}
 & & \downarrow & & \downarrow & & \downarrow & & \\
\rightarrow & & HF_{*,(k-1,l-1)} & \rightarrow & HF_{*,(k,l-1)} & \rightarrow & HF_{*,(k+1,l-1)} & \rightarrow & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
(3.1) \quad \rightarrow & & HF_{*,(k-1,l)} & \rightarrow & HF_{*,(k,l)} & \stackrel{\Phi_{(k,l)}}{\rightarrow} & HF_{*,(k+1,l)} & \rightarrow & \\
 & & \downarrow & & \downarrow \Psi_{(k,l)} & & \downarrow & & \\
\rightarrow & & HF_{*,(k-1,l+1)} & \rightarrow & HF_{*,(k,l+1)} & \rightarrow & HF_{*,(k+1,l+1)} & \rightarrow & \\
 & & \downarrow & & \downarrow & & \downarrow & &
\end{array}
$$

We define the modified Floer homology group as follows:

$$\widehat{HF}_* := \varinjlim_{l \to +\infty} \varprojlim_{k \to -\infty} HF_{*,(k,l)}.$$

It is easy to see

Lemma 3.2 *For a generic pair (H, J), $\widehat{HF}_*(H, J)$ does not depend on the choice of $\{r_j\}$.*

As a module, Λ_ω is the completion of the group ring $\mathbf{Z}/2[\Gamma]$ with respect to the following filtration:

$$\mathbf{Z}/2[\Gamma]_{(i,j)} := \{\sum_{A \in \Gamma} \lambda_A \cdot \delta_A \in \mathbf{Z}/2[\Gamma] \mid \lambda_A = 0 \text{ for } \phi_\omega(A) > -i \text{ or } \phi_\omega(A) < -j\}$$

$$\Lambda_\omega = \varinjlim_{j \to +\infty} \varprojlim_{i \to -\infty} \mathbf{Z}/2[\Gamma]_{(i,j)}.$$

For a generic Hamiltonian function H, $\mathcal{P}(H)$ is a finite set. Since $[\omega]$ is an integral class, we can choose the set $\{r_j\} = \{\frac{j}{q} + \epsilon | j \in \mathbf{Z}\}$ for some positive integer q and some $\epsilon \geq 0$. The Γ-action on $(C_*(H, J), \partial)$ satisfies the following

$$\mathbf{Z}/2[\Gamma]_{(i,j)} \times HF_{*,(k,l)}(H, J) \to HF_{*,(k+iq,l+jq)}(H, J)),$$

which induces Λ_ω- action on $\widehat{HF}_*(H, J)$. Thus we get

Lemma 3.3 *$\widehat{HF}_*(H, J)$ has a natural Λ_ω-module structure.*

The following theorem is an analogue of Theorem (2.8).

Theorem 3.4 *For generic pairs* (H^α, J^α) *and* (H^β, J^β)*, there exists a* Λ_ω*-module isomorphism*

$$HF^{\beta\alpha} : \widehat{HF}_*(H^\alpha, J^\alpha) \to \widehat{HF}_*(H^\beta, J^\beta)$$

presering the Conley-Zehnder index. If (H^γ, J^γ) *is any other such pair then*

$$HF^{\gamma\beta} \circ HF^{\beta\alpha} = HF^{\gamma\alpha}, HF^{\alpha\alpha} = id.$$

Proof. Let $\{(H(\sigma), J(\sigma)) \mid \sigma \in [0,1]\}$ be a path connecting (H^α, J^α) and (H^β, J^β). Subdivide $[0,1]$ into $[s_k, s_{k+1}]$ such that there exists $\{r_j^{(k)}\}$ and $\epsilon > 0$ satisfying

(i) $r_j^{(k)} \to \pm\infty$ as $j \to \pm\infty$.

(ii) ϵ-neighborhood of $\{r_j^{(k)}\}$ contains no critical values of a_{H_s} for $s = s_k, s_{k+1}$.

(iii) $\displaystyle\int_{s_k}^{s_{k+1}} \max_{x \in M, t \in S^1} \left|\frac{\partial}{\partial s} H_s(t,x)\right| ds < \epsilon.$

This is possible, since $[\omega]$ is an integral class and $\mathcal{P}(H(\sigma))$ is finite for $\sigma \in [0,1]$ (see Lemma (3.5) below). Let $\{(H_s, J_s) \mid s \in \mathbf{R}\}$ be a generic path satisfying (2.9) with $(H^\alpha, J^\alpha) = (H_{s_k}, J_{s_k})$ and $(H^\beta, J^\beta) = (H_{s_{k+1}}, J_{s_{k+1}})$. We shall consider the equation (2.10) with $a_{H^\alpha}(\tilde{z}), a_{H^\beta}(\tilde{w}) \in [r_i^{(k)}, r_j^{(k)}]$. Let $\{u_l\}$ be a sequence of solutions. Since we have a uniform upper bound for the energy functional (2.14), $\{u_l\}$ contains a subsequence which converges to a solution of (2.10) with $\tilde{z}' \in \tilde{\mathcal{P}}(H^\alpha)$ and $\tilde{w}' \in \tilde{\mathcal{P}}(H^\beta)$, and possibly solutions of (2.3) with $H = H^\alpha$, $\tilde{z}', \tilde{z}'' \in \tilde{\mathcal{P}}(H^\alpha)$ or $H = H^\beta$, $\tilde{w}', \tilde{w}'' \in \tilde{\mathcal{P}}(H^\beta)$. The condition (iii) implies that $a_{H^\alpha}(\tilde{z}'), a_{H^\alpha}(\tilde{z}''), a_{H^\beta}(\tilde{w}'), a_{H^\beta}(\tilde{w}'') \in [r_i^{(k)}, r_j^{(k)}]$. Hence the proof of Theorem (2.8) yields that $\phi^{\beta\alpha}$ induces a chain mapping $C_{*,(i,j)}(H^\alpha, J^\alpha) \to C_{*,(i,j)}(H^\beta, J^\beta)$. In a similar way, we can show that $\phi^{\beta\alpha}$ does not depend on the choice of $\{H_s, J_s\}$ and that $\phi^{\beta\alpha}$ induces an isomorphism between "relative" Floer homology groups. It is also easy to see that homomorphisms $\Phi_{(i,j)}, \Psi_{(i,j)}$ in the diagram (3.1) are compatible with the isomorphism obtained above. Moreover, the actions of Λ_ω are preserved under the induced isomorphism between modified Floer homology groups $\widehat{HF}_*(H^\alpha, J_\alpha)$ and $\widehat{HF}_*(H^\beta, J^\beta)$. □

Lemma 3.5 *For a generic Hamiltonian functions H^α and H^β, there is a path $\{H(s)\}$ connecting them such that $\mathcal{P}(H(s))$ is finite for all s.*

Proof. Let $\mathcal{H}$ be the Banach space of periodic Hamiltonian functions (see [H-S]) and $\{H_s\}$ a generic path in $\mathcal{H}$ connecting H^α and H^β. The implicit function theorem and the Sard-Smale theorem [Sm] yield that $\mathcal{X}(\{H_s\}) = \{(x,s) \in \mathcal{L}(M) \times [0,1] \mid x \in \mathcal{P}(H_s)\}$ is a 1-dimensional manifold with boundary $\mathcal{P}(H_0) \times \{0\} \cup \mathcal{P}(H_1) \times \{1\}$.

More precisely, let $\mathcal{E}$ denotes the Banach space bundle over $W^{1,2}$-completion of $\mathcal{L}(M)$, which we shall also denote by $\mathcal{L}(M)$ with fiber $\mathcal{E}_x = L^2\Gamma(x^*TM)$. We define a Fredholm mapping $F : \mathcal{L}(M) \times \mathbf{R} \to \mathcal{E}$ by

$$F(x,s) = \dot{x} + X_{H_s}(t, x(t)).$$

Then the linearization DF of F is

$$DF(\xi,\sigma) = \nabla_{\frac{\partial}{\partial t}}\xi + \nabla_\xi X_{H_s}(t,x(t)) + \sigma X_{\frac{\partial}{\partial s}H_s}(t,x(t))$$

for $(\xi,\sigma) \in T_{(x,s)}(\mathcal{L}(M) \times \mathbf{R})$. For a generic path $\{H_s\}$, DF is surjective, this fact and the index computation imply that $\mathcal{X}(\{H_s\})$ is a 1-dimensional manifold.

Let $p : \mathcal{X} \to [0,1]$ be the projection to the second factor. To prove Lemma (3.5), it suffices to show that $dp : T\mathcal{X} \to T([0,1])$ is transversal to the zero section of $T([0,1])$ outside of the zero section of $T\mathcal{X}$. Namely we have to get the transversality on 1-jets. We define $\mathcal{F} : \mathcal{L}(M) \times \mathbf{R} \times \mathcal{H} \to \mathcal{E}$ by

$$\mathcal{F}(x,s,H) = \dot{x} + X_{H_s+H}(t,x(t)).$$

Restricting the linearization of $\mathcal{F}$ to $T(\mathcal{L}(M) \times \mathbf{R}) \times \mathcal{H}$, we get $\mathcal{F}' : T(\mathcal{L}(M) \times \mathbf{R}) \times \mathcal{H} \to T\mathcal{E}$ as follows:

$$\mathcal{F}'(x,s,\xi,\sigma,H) = (\mathcal{F}'_{(1)}(x,s,\xi,\sigma,H), \mathcal{F}'_{(2)}(x,s,\xi,\sigma,H)),$$

where

$$\mathcal{F}'_{(1)}(x,s,\xi,\sigma,H) = \dot{x} + X_{H_s+H}(t,x(t)),$$
$$\mathcal{F}'_{(2)}(x,s,\xi,\sigma,H) = \nabla_{\frac{\partial}{\partial t}}\xi + \nabla_\xi X_{H_s+H}(t,x(t)) + \sigma X_{\frac{\partial}{\partial s}H_s}(t,x(t))$$

for $(\xi,\sigma) \in T_{(x,s)}(\mathcal{L}(M) \times \mathbf{R})$, $H \in \mathcal{H}$. The linearization $D\mathcal{F}'$ of $\mathcal{F}'$ is given by

$$D\mathcal{F}'(a,b,c,\tau,h) = (D\mathcal{F}'_{(1)}(a,b,c,\tau,h), D\mathcal{F}'_{(2)}(a,b,c,\tau,h))$$

where

$$D\mathcal{F}'_{(1)}(a,b,c,\tau,h) = \nabla_{\frac{\partial}{\partial t}} a + \nabla_a X_{H_s+H}(t,x(t)) + bX_{\frac{\partial}{\partial s}H_s}(t,x(t)) + X_h(t,x(t)),$$

and

$$\begin{aligned} D\mathcal{F}'_{(2)}(a,b,c,\tau,h) &= \nabla_{\frac{\partial}{\partial t}} c + \nabla_c X_{H_s+H}(t,x(t)) + \tau X_{\frac{\partial}{\partial s}H_s}(t,x(t)) \\ &\quad + \nabla_\xi X_h(t,x(t)) + b\nabla_\xi X_{\frac{\partial}{\partial s}H_s}(t,x(t)) \\ &\quad + b\sigma X_{\frac{\partial^2}{\partial s^2}H_s}(t,x(t)) + \nabla_a \nabla_\xi X_{H_s+H}(t,x(t)) \\ &\quad + \sigma \nabla_a X_{\frac{\partial}{\partial s}H_s}(t,x(t)). \end{aligned}$$

$p_4 : T(\mathcal{L}(M)\times \mathbf{R})\times\mathcal{H} \to \mathbf{R}$ denotes the projection to the fourth factor. For H sufficiently close to 0, $(a,b)\mapsto \nabla_{\frac{\partial}{\partial t}} a + \nabla_a X_{H_s+H}(t,x(t)) + bX_{\frac{\partial}{\partial s}H_s}(t,x(t))$ is surjective. Because of the term $\nabla_\xi X_h(t,x(t))$ and the unique continuation theorem, $(c,h) \mapsto \nabla_{\frac{\partial}{\partial t}} c + \nabla_c X_{H_s+H}(t,x(t)) + \nabla_\xi X_h(t,x(t))$ is surjective. Hence, $\mathcal{F}'\times p_4$ is transversal to the zero section at points $(x,s,\xi,0,H)\in T(\mathcal{L}(M)\times\mathbf{R})\times\mathcal{H}$ satisfying $\xi\neq 0$. By the Sard-Smale theorem, $\{(x,s,\xi,0)\in T(\mathcal{L}(M)\times\mathbf{R}) \mid x\in\mathcal{P}(H_s+H), \xi\in T_{(x,s)}\mathcal{X}(\{H_s+H\}), \xi\neq 0\}$ is a 1-dimensional manifold for a generic H. In particular, $\mathcal{C} = \{(x,s)\in\mathcal{L}(M)\times\mathbf{R} \mid x\in\mathcal{P}(H_s+H), p_{4*}(\xi)=0 \text{ for all } \xi\in T_{(x,s)}\mathcal{X}(\{H_s+H\})\}$ is a 0-dimensional submanifold of $\mathcal{X}(H_s+H)$. By the assumption on H^α and H^β, $\mathcal{C}$ does not intersect the boundary of $\mathcal{X}(\{H_s+H\})$ for H sufficiently close to 0. Moreover, there is a path $\gamma:[0,1]\to\mathcal{H}$ such that $\gamma(s)=0$ near $s=0$, $\gamma(s)=H$ near $s=1$, $\mathcal{X}(\{\gamma+H^{\alpha \text{ or } \beta}\}) = \{(x,s)\mid x\in\mathcal{P}(\gamma(s)+H^{\alpha\text{ or }\beta})\}$ is a manifold, and the projection to the second factor $\mathcal{X}(\gamma+H^{\alpha\text{ or }\beta})\}\to[0,1]$ is a submersion. Then we define $H(s) = H^\alpha + \gamma(\epsilon^{-1}\cdot s)$ for $0\le s\le\epsilon$, $H(s) = H_{(1-2\epsilon)^{-1}(s-\epsilon)} + H$ for $\epsilon\le s\le 1-\epsilon$, and $H(s) = H^\beta + \gamma(\epsilon^{-1}(1-s))$ for $1-\epsilon\le s\le 1$. This satisfies the property of Lemma (3.5). □

4 Computation of the modified Floer homology group.

In [F],[H-S], they compare the Floer complex of a generic pair (H,J) with the Morse complex of a Morse function. An almost complex structure J calibrated by ω determines a Riemannian metric on M. For a Morse function

$f : M \to \mathbf{R}$ whose gradient flow is of Morse-Smale type, $C_*(f)$ denotes the Morse complex associated to f [Sa].

Under the assumption that (M, ω) is monotone or $c_1(M)(\pi_2(M)) = 0$, or the minimal Chern number $N \geq n$, they proved that $HF_*(f, J)$ is isomorphic to $H_{*+n}(C_*(f)) \otimes \Lambda_\omega \cong H_{*+n}(M; \mathbf{Z}/2) \otimes \Lambda_\omega$ as graded Λ_ω -modules. Here $H_*(M; \mathbf{Z}/2) \otimes \Lambda_\omega$ is the tensor product of graded modules. This result and Theorem(2.8) yield Theorem(2.15).

In this section, we compute the modified Floer homology group of (H, J) without the assumption concerning the minimal Chern number.

First of all, we show the following

Lemma 4.1 *For a fixed $C > 0$, there exists a positive integer $j_0(C)$ such that for $\tilde{x}, \tilde{y} \in \tilde{\mathcal{P}}(f)$ satisfying $a_f(\tilde{x}) - a_f(\tilde{y}) < C, \mu_f(\tilde{x}) - \mu_f(\tilde{y}) \leq 1$, all solutions of the following equation with $\epsilon = 1/j$ are independent of t-variable for $j > j_0(C)$ and a generic almost complex structure J.*

$$\frac{\partial u}{\partial s} + J(u)\frac{\partial u}{\partial t} + \epsilon \cdot (\nabla f)(u) = 0 \tag{4.2}$$

$$\lim_{s \to -\infty} u(s,t) = \tilde{x}(t), \lim_{s \to +\infty} u(s,t) = \tilde{y}(t) \tag{4.3}$$

Proof. First of all, we show that $c_1(u) < 0$. Suppose that $c_1(u) \geq 0$. Let $\mathrm{ind}_f(x)$ denote the index of the Hessian of f at a critical point x. Then we have $\mu_f(\tilde{x}) - \mu_f(\tilde{y}) = \mathrm{ind}_f(x) - \mathrm{ind}_f(y) + 2c_1(u)$. If u is somewhere injective, $\mathcal{M}(\tilde{x}, \tilde{y})$ is a manifold of dimension $\mu_f(\tilde{x}) - \mu_f(\tilde{y})$ around u for a generic almost complex structure J. On the other hand, there is a 2-parameter family of solutions $u_{\sigma,\rho}(s,t) = u(s + \sigma, t + \rho)$ in $\mathcal{M}(\tilde{x}, \tilde{y})$. Hence $\mu_f(\tilde{x}) - \mu_f(\tilde{y}) = \dim \mathcal{M}(\tilde{x}, \tilde{y}) \geq 2$. If u is not somewhere injective but depending on t-variable, there exist a positive interger n and a solution v of the equation (4.2), (4.3) replacing ϵ by ϵ/n such that $u(s,t) = v(ns, nt)$. The above argument shows that $\mathrm{ind}_f(x) - \mathrm{ind}_f(y) + 2c_1(v) \geq 2$. Since $c_1(u) = n \cdot c_1(v)$ and $c_1(u) \geq 0$, $\mu_f(\tilde{x}) - \mu_f(\tilde{y}) = \mathrm{ind}_f(x) - \mathrm{ind}_f(y) + 2c_1(u) \geq 2$. It contradicts the assumption that $\mu_f(\tilde{x}) - \mu_f(\tilde{y}) \leq 1$.

From now on, we assume that $c_1(u) < 0$. If the statement is false, we can choose a sequence of integers j_l diverging to $+\infty$ and a sequence of solutions

u_l of (4.2), (4.3) with $\epsilon = 1/j_l$. Since the energy of u_l is uniformly bounded by C, u_l converges, up to J-holomorphic bubbles, to a solution u_∞ of (4.2) with $\epsilon = 0$, i.e. J-holomorphic mapping and u_∞ extends to a J-holomorphic mapping from the Riemann sphere to M. u_∞ may be a constant mapping. If $\epsilon \neq 0$, only constant solutions of (4.2) are constant mappings to critical points of f. On the other hand, every constant mapping is a solution of (4.2) with $\epsilon = 0$. For real numbers σ_l, ψ_l denotes the reparametrization $(s,t) \to (s + \sigma_l, t)$ of the infinite cylinder $\mathbf{R} \times S^1$. The above argument yields that $u_l \circ \psi_l$ converges to a J-holomorphic sphere up to J-holomorphic bubbles. Let $\{S_j\}$ be the set of all possible J-holomorphic spheres appearing as a limit of $u_l \circ \psi_l$ or bubbles.

Claim 1. $\{S_j\}$ is not an empty set.

Proof of Claim 1. Let δ be the injectivity radius of M and $v : \mathbf{R} \times S^1 \to M$ a smooth mapping satisfying the following asymptotic condition.

$$\lim_{s \to -\infty} v(s,t) = x, \lim_{s \to +\infty} v(s,t) = y.$$

We also denote by v the extension of v to $S^2 \to M$. If v satisfies $|v_*(s,t)(\frac{\partial}{\partial t})| \leq \delta$ for all $(s,t) \in \mathbf{R} \times S^1$, v is homologous to zero. Since $c_1(u_j) < 0$, there exist (s_j, t_j) such that $|u_{j*}(s_j,t_j)(\frac{\partial}{\partial t})| > \delta$. We reparametrize u_j by $u'_j(s,t) = u_j(s + s_j, t + t_j)$, then u'_j is still a solution of (4.2) with $\epsilon = 1/j$, and satisfies

$$\left| u'_{j*}(0,0)(\frac{\partial}{\partial t}) \right| > \delta \tag{4.4}$$

Since we have a uniform energy bound, u'_j converges to a J-holomorphic sphere possibly with J-holomorphic bubbles, which are also J-holomorphic spheres. The condition (4.4) assures that at least one of the J-holomorphic spheres above is not a constant mapping.

Claim 2. $\{S_j\}$ is a finite set.

Proof of Claim 2. Let $\{T_i\}$ be J-holomorphic spheres obtained as limits of solutions of (4.2) with $\epsilon = 1/j$ except finitely many points in $\mathbf{R} \times S^1$ and $\{T_{i,l} \mid 1 \leq l \leq d(i)\}$ J-holomorphic bubbles attached to T_i. For any $\varepsilon > 0$, we can take j large enough such that there exist real numbers R_i, L_i $(i = 1, \cdots, k)$ such that $[R_i, L_i]$ $(i = 1, \cdots, k)$ are mutually disjoint and

$u_j([R_i, L_i] \times S^1)$ is close to T_i and possibly some bubbles $T_{i,l}$ enough to satisfy

$$\frac{1}{2}\int_{R_i}^{L_i}\int_0^1 \left|\frac{\partial u_j}{\partial s}\right|^2 + \left|\frac{\partial u_j}{\partial t} + \frac{1}{j}\cdot X_f\right|^2 dtds \geq E(T_i) + \sum_{l=1}^{d(i)} E(T_{i,l}) - \varepsilon.$$

Hence

$$E(u_j) \geq \sum_{i=1}^{k}\{E(T_i) + \sum_{l=1}^{d(i)} E(T_{i,l})\} - k\varepsilon,$$

if $\{T_i\}$ contains at least k J-holomorphic spheres. On the other hand, we have $E(u_j) \leq C$ and $E(S) = \int_S \omega \geq 1$, because $[\omega]$ is an integral class. Since ε is arbitrary, the cardinality of $\{S_i\} = \{T_i, T_{i,l}\}$ is bounded by C.

Remark. Hofer and Salamon showed the estimate $E(S) > \hbar$ for some positive constant $\hbar$ without assuming $[\omega]$ is an integral class.

Claim 3. $c_1(u_l) = \sum_j c_1(S_j)$.

Proof of Claim 3. Let U be a regular neighborhood of $\bigcup\{T_i \cup (\cup T_{i,l})\}$. For a fixed $\varepsilon > 0$, there exists j and sequence of real numbers $-\infty = L_0 < R_1 < L_1 < R_2 < L_2 < \cdots < R_k < L_k < R_{k+1} = +\infty$, such that

$$\mathrm{Im}\ u_j([R_i, L_i] \times S^1) \subset U,$$

and

$$\left|\frac{\partial}{\partial t} u_j(s,t)\right| < \varepsilon \text{ if } s < [L_i, R_{i+1}] \text{ for some } i = 0, \cdots k. \tag{4.5}$$

We choose $\varepsilon < \delta$, then $u_j|_{R_i}$, $u_j|_{L_i}$ bound disks D_i^-, D_i^+ in δ-balls, which are unique up to homotopy. It is easy to see that $C_i = D_i^- \cup u_j([R_i, L_i] \times S^1) \cup D_i^+$ is homologous to $T_i \cup (\cup T_{i,l})$. The condition (4.5) assures that $D_{i-1}^+ \cup u_j([L_{i-1}, R_i] \times S^1) \cup D_i^-$ is homologous to zero. Therefore we get

$$c_1(u_j) = \sum c_1(C_i) = \sum (c_1(T_i) + \sum c_1(T_{i,l})).$$

Since $c_1(u_l) < 0$, one of the J-holomorphic spheres S_j has negative Chern number. However the weak monotonicity excludes this possibility for a generic almost complex structure J. This is a contradiction. □

We choose a C^2-small Morse function f satisfying $-1/8 < f(x) < 1/8$ and $\{r_j\} = \{j + 1/2 | j \in \mathbf{Z}\}$. Let $\{(H_s, J_s) | 0 \leq s \leq 1\}$ be a generic path from (f, J) to a generic pair (H_1, J_1), which is sufficiently small in C^1-sense. More precisely, the set of critical values of a_{H_s} is disjoint from $\{j + 1/2 + \delta \mid j \in \mathbf{Z}, -1/16 < \delta < 1/16\}$ and

$$\left| \frac{\partial}{\partial s} H_s(t, x) \right| < \frac{1}{16}$$

for all $x \in M$ and $s \in [0, 1]$. We prove the following

Theorem 4.6 $\widehat{HF}_*(H_1, J_1) \cong H_{*+n}(M; \mathbf{Z}/2) \otimes \Lambda_\omega$.

Proof. Let $\tilde{x}, \tilde{y} \in \tilde{\mathcal{P}}(f)$ satisfy $k + 1/2 < a_f(\tilde{x}), a_f(\tilde{y}) < l + 1/2$ and $\mu_f(\tilde{x}) - \mu_f(\tilde{y}) \leq 1$. By Lemma (4.1), there exists a positive integer $j_0(k, l)$ such that all solutions of the equation (4.2) with $\epsilon = 1/j$ are independent of t-variable if $j > j_0(k, l)$. In particular, the equation (4.2) has no non-trivial solutions if $\mu_f(\tilde{x}) = \mu_f(\tilde{y})$, and the chain complex $C_{*,(k,l)}(1/j \cdot f, J)$ is isomorphic to $C_{*+n}(f) \otimes \mathbf{Z}/2[\Gamma]_{(k,l)}$. The argument in the proof of Theorem (3.4) yields a chain mapping $\phi_{(k,l)} : C_{*,(k,l)}(H, J) \to C_{*+n,(k,l)}(1/j \cdot f, J) \cong C_{*+n}(f) \otimes \mathbf{Z}/2[\Gamma]_{(k,l)}$, which induces an isomorphism between homology groups. Moreover the argument in the proof of Lemma (4.1) yields that there exists an integer $j_1(k, l) \geq j_0(k, l)$ such that all the solutions of the equation below

$$\frac{\partial u}{\partial s} + J(u) \frac{\partial u}{\partial t} + \varphi(s)(\nabla f)(u) = 0$$

$$\lim_{s \to -\infty} u(s, t) = x \in \mathcal{P}(f), \ \lim_{s \to \infty} u(s, t) = y \in \mathcal{P}(f)$$

$$(y, u^- \sharp u) \sim (y, u^+)$$

are t-independent, if $\mu(\tilde{x}) = \mu(\tilde{y})$, where $\tilde{x} = (x, u^-)$, $\tilde{y} = (y, u^+)$, and a function $\varphi(s)$ on $\mathbf{R}$ satisfies

$$|\varphi(s)| \leq \frac{1}{j_1(k, l)},$$

and

$$\varphi(s) = \frac{1}{j_1} \text{ for } s < -R, \ \varphi(s) = \frac{1}{j_2} \text{ for } s > R,$$

for some $R > 0$. t-independent solutions of the above equation are reparametrized paths of the gradient trajectory of f. Since we assume that the gradient flow is of Morse-Smale type, gradient trajectories are constant paths at critical points of f, if the Morse indices at end points coincide. This observation implies that the induced homomorphism $\phi_{(k,l)_*}$ between homology groups does not depend on the choice of $j > j_1(k,l)$. Therefore homomorphisms $\{\phi_{(k,l)_*}\}$ commute with homomorphisms $\Phi_{(k,l)}, \Psi_{(k,l)}$ in the diagram (3.1). Thus $\{\phi_{(k,l)_*}\}$ induces a homomorphism $\phi : \widehat{HF}_*(H,J) \to H_{*+n}(M;\mathbf{Z}/2) \otimes \Lambda_\omega$. Since $\{\phi_{(k,l)_*}\}$ are isomorphisms, ϕ is an isomorphism. By the construction, it is easy to see that ϕ is Λ_ω-linear. □

Theorem (3.4) and Theorem (4.6) yield

Theorem 4.7 *Let (M,ω) be a weakly monotone symplectic manifold such that $[\omega] \in Im(H^2(M;\mathbf{Z}) \to H^2(M,\mathbf{R}))$. Then for a generic pair (H,J),*

$$\widehat{HF}_*(H,J) \cong H_{*+n}(M;\mathbf{Z}/2) \otimes \Lambda_\omega.$$

Theorem (1.1) is equivalent to the following

Corollary 4.8 *Let (M,ω) be a weakly monotone symplectic manifold and H a Hamiltonian function such that all periodic solutions of (2.1) are non-degenerate. The number of periodic solutions of (2.1) is at least $\sum_p b_p(M;\mathbf{Z}/2)$, where $b_p(M;\mathbf{Z}/2)$ is the p-th $\mathbf{Z}/2$-Betti number of M.*

Proof. If $[\omega]$ is an integral class, the conclusion is a direct consequence of Theorem (4.7). The same conclusion holds if $[\omega]$ is in $H^2(M;\mathbf{Q})$. We shall show that the general case is reduced to this case. Let $\{x_i\}$ be all periodic solutions of (2.1), $N_i(\epsilon)$ an ϵ-neighborhood of the orbit of x_i, and ϕ_i a cut off function, i.e. $\phi_i = 1$ on $N_i(\epsilon/2)$ and $\phi_i = 0$ outside of $N_i(3\epsilon/4)$. $\eta_1, \cdots, \eta_q$ denote closed 2-forms on M representing generators of $H^1(M;\mathbf{R})$. For a sufficiently small ϵ, $N_i(\epsilon)$ has the same homotopy type as the orbit of x_i, hence $H^2(N_i(\epsilon);\mathbf{R}) = 0$. Thus $\eta_j \mid_{N_i(\epsilon)} = dg_{j,i}$ for some function $g_{j,i}$ on $N_i(\epsilon)$. $\eta'_j = \eta_j - d(\sum_i \phi_i \cdot g_{j,i})$ is cohomologous to η_j, with support in $M - \cup_i N_i(\epsilon/2)$. It is easy to see that there exists $\sigma > 0$, such that the equation (2.1) has exactly same number of solutions for symplectic forms $\omega' = \omega + \sum a_k \cdot \eta'_k$ if $|a_k| < \sigma$. Since $H^2(M;\mathbf{Q})$ is dense in $H^2(M;\mathbf{R})$, there exist real numbers

a_k such that $\omega' \in H^2(M;\mathbf{Q})$ and $|a_k| < \sigma$. Moreover, there are no J-holomorphic spheres for a generic J calibrated by ω' and tamed by ω. This condition is the only one we use in the proof of Theorem (4.7). Therefore we get the desired estimate. □

Since the weak monotonicity is automatic in dimension 2, 4 and 6, we get

Corollary 4.9 *Let* (M,ω) *be a closed symplectic manifold of dimension* 2,4 *or* 6. *If all periodic solutions of (2.1) are non-degenerate, the number of periodic solutions is at least* $\sum b_p(M;\mathbf{Z}/2)$.

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