ON THE HOMEOMORPHISM CLASSIFICATION OF SMOOTH KNOTTED SURFACES IN THE 4–SPHERE

by

Matthias Kreck

Max–Planck–Institut
für Mathematik
Gottfried–Claren–Str. 26
5300 Bonn 3
Federal Republic of Germany

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1. In [FKV] an infinite family of smooth (real) surfaces F_k embedded in S^4 was constructed which has the following properties:

i) The knottings (S^4,F_k) and (S^4,F_ℓ) are not diffeomorphic for $k \neq \ell$.

ii) $F_k = \#10(\mathbb{R}P^2)$

iii) $\pi_1(S^4 - F_k) = \mathbb{Z}_2$

iv) The normal Euler number (with local coefficients) of F_k in S^4 is 16.

The knottings (S^4,F_k) are constructed from the Dolgachev surfaces $D_{2,2k+1}$. There are antiholomorphic involutions c on $D_{2,2k+1}$ with fixed point set $F_k = \#10(\mathbb{R}P^2)$ and orbit space $D_{2,2k+1}/c$ diffeomorphic to S^4. Thus the diffeomorphism type of $D_{2,2k+1}$, the ramified covering along the knotting, is an invariant and one can distinguish these Dolgachev surface by Donaldson's Γ–type invariants [D], [FM], [OV]. It was also proved in [FKV] that the number of homeomorphism types of these knottings is finite and it was conjectured that they are all homeomorphic to the standard embedding (S^4,F) with normal Euler number 16. The main result of this note is an affirmative answer to this conjecture.

More precisely consider the standard embedding of $\mathbb{R}P^2$ into S^4 with normal Euler class –2. This can be considered as the fixed point set of the standard antiholomorphic involution c on $\mathbb{C}P^2$ embedded into $\mathbb{C}P^2/c \cong S^4$. Then the standard embedding (S^4,F) with normal Euler class 16 is obtained by taking the connected sum $(S^4,\mathbb{R}P^2)\#9(-S^4,\mathbb{R}P^2)$.

Theorem: Let $S = \#10(\mathbb{R}P^2)$ be embedded into S^4 with normal Euler number 16 and $\pi_1(S^4-S) = \mathbb{Z}_2$. Then (S^4,S) is homeomorphic to (S^4,F), the standard embedding with normal Euler number 16. The homeomorphism can be chosen as a diffeomorphism on a neighborhood of S and F.

Corollary: The knottings (S^4,F_k) are all homeomorphic to (S^4,F) implying that the standard knotting (S^4,F) has infinitely many smooth structures.

Remark: Recently R. Gompf [G] constructed non–diffeomorphic embeddings of a punctured Klein bottle K (= Klein bottle minus open 2–ball) into D^4 with $\pi_1(D^4 - K) = \mathbb{Z}_2$ and intersection form of the 2–fold ramified covering along K equal to $<1> \oplus <-1>$. The same methods as used for the proof of our Theorem show that they are pairwise homeomorphic if they have same relative normal Euler number and the knots ∂K in S^3 are equal. We will comment the necessary modifications of the proof in section 5. I was informed by O. Viro that he has similar knottings of K in D^4 which are related to the construction in [V].

2. Proof: Since F and S have isomorphic normal bundles we can choose a linear identification of open tubular neighborhoods and denote the complements by C and C'. We identify the boundaries, so that $\partial C = \partial C' =: M$. We want to extend the identity on M to a homeomorphism from C to C'. Since C and C' are Spin–manifolds a necessary condition for this is that we can choose Spin–structures on C and C' which agree on the common boundary. Another necessary condition is that the diagram

$$\begin{array}{ccc} \pi_1(M) & \longrightarrow & \pi_1(C) \\ \downarrow & & \downarrow \\ \pi_1(C') & \longrightarrow & \mathbb{Z}/2 \end{array} \tag{1}$$

commutes. One can show that by choosing the linear identification of the tubular neighborhoods appropriately one can achieve these two necessary conditions. I am indepted to O. Viro for this information. To obtain condition (1), choose section s and s' from F^o resp. S^o (delete an open 2–disk) to M such that the composition with the inclusion to C and C' resp. are trivial on π_1. Since the normal Euler numbers of the knottings are equal one can choose the linear identification of the tubular neighborhoods such that they commute with s and s' resp. yielding (1). To obtain the compatibility of Spin structures on M it is enough to control them on the image of s and s'. Note that for each embedded circle α in F^o, $s(\alpha)$ bounds an immersed disk D in C. The normal bundle of α determines a 1–dimensional subbundle of $\nu(D)|_{\partial D}$. The Spin structure on the image of s is characterized by the obstruction mod 4 to extending this subbundle to $\nu(D)$ and gives a quadratic form $q : H_1(F^o) \longrightarrow \mathbb{Z}/4\mathbb{Z}$ [GM]. Thus we have to control that the identification of F and S respects this form or equivalently that the Brown invariants in $\mathbb{Z}/8\mathbb{Z}$ agree. But this follows from the generalized Rochlin formula [GM].

In the following we will assume that the Spin–structures on $\partial C = \partial C' = M$ agree and the diagram (1) commutes. There is another obvious invariant to be controlled, the intersection form on the universal covering. For this we assign to our knotted surface the 2–fold ramified covering along F denoted by X. A simple calculation shows that X is 1–connected, $e(X) = 12$ and $\mathrm{sign}(X) = -8$. Thus the intersection form on X is indefinite and odd (since otherwise the signature were divisible by 16 by Rochlin's Theorem). By the classification of indefinite forms, the intersection form on X is $<1> \oplus 9 <-1>$. The long exact homology sequence combined with excision and Poincaré duality leads to an exact sequence

$$0 \longrightarrow H^1(F) \longrightarrow H_2(\breve{C}) \longrightarrow H_2(X) \longrightarrow \underset{\mathbb{Z}/2}{\overset{\|}{H^2(F)}} \longrightarrow 0$$

and the map $H_2(X) \longrightarrow H^2(F) = \mathbb{Z}_2$ is $\alpha \longmapsto \alpha \circ [F]$, the mod 2 intersection number of α with F. Since $\breve{C}$ is Spin and X is not Spin (see above) the map $\alpha \longmapsto \alpha \circ [F]$ is given by $w_2(X) : \alpha \circ [F] = <w_2(X),\alpha>$. Since the image of $H^1(F)$ is contained in the radical of the intersection form on $H_2(\breve{C})$ and the form on $H_2(X)$ restricted to the kernel of w_2 is non–singular, the image of $H^1(F)$ is the radical of the form on $H_2(\breve{C})$. The form on $H_2(\breve{C})/\mathrm{rad}$ is the restriction of $<1> \oplus 9<-1> \cong E_8 \oplus <1> \oplus <-1>$ to the kernel of $x \longmapsto x \circ x$ which is $E_8 \oplus \begin{bmatrix} 4 & 2 \\ 2 & 0 \end{bmatrix} \cong E_8 + 2\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

We know that the covering transformation τ acts trivially on $H^1(F)$ and by -1 on $H_2(X)$ (since $X/\tau = S^4$). Thus, if we take the $\Lambda = \mathbb{Z}[\mathbb{Z}_2]$ module structure given by τ on $H_2(\breve{C})$ into account we have an exact sequence

$$0 \longrightarrow \mathbb{Z}^9_+ \longrightarrow H_2(\breve{C}) \longrightarrow \mathbb{Z}^{10}_- \longrightarrow 0$$

where + or – indicates the trivial or non–trivial Λ–action. Moreover one can show that $H_2(\breve{C}) = \mathbb{Z}_- \oplus \Lambda^9$ ([FKV], Lemma 5.2A). We can summarize these considerations as follows:

$H_2(\tilde{C}) \cong \mathbb{Z}_- \oplus \Lambda^9$;

(2) the radical of the intersection form is $H_2(\tilde{C})_+$, the $+1$ eigenspace; the form on $H_2(\tilde{C})/_{\text{rad}}$ is $E_8 \oplus 2\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

The proof is finished by the following proposition which is the main step.

Proposition: Let C and C' be 4–dimensional Spin manifolds with fundamental group $\mathbb{Z}_2$, $\partial C = \partial C' = M$ and inducing same Spin–structure on M such that the conditions (1) and (2) are fulfilled. Then there is a homeomorphism from C to C' inducing on M the identity.

3. Proof of the Proposition. We use the method of [K]. The normal 1–type of C is the fibration $p : B = \mathbb{R}P^\infty \times B\,\text{Spin} \xrightarrow{p_2} BO$ and a normal smoothing of X in (B,p) is given by the non–trivial map $C \longrightarrow \mathbb{R}P^\infty$ and a Spin–structure on C (given by a lift of the normal Gauß map to B Spin). Thus it is uniquely determined by a Spin–structure. By assumption there exist normal smoothings of C and C' in (B,p) which agree on the common boundary. Thus we can form $C \cup (-C')$, a closed manifold with (B,p)–structure. An easy computation with the Atiyah-Hirzebruch spectral sequence shows that $\Omega_4(B,p) \cong \mathbb{Z}$, detected by the signature. Since sign C = sing C', $C \cup -C'$ is zero bordant in (B,p).

Let W be a zero bordism. Then there exists an obstruction $\Theta(W,C) \in \ell_5(\mathbb{Z}/2)$ such that C is h–cobordant to C' rel. boundary if and only if $\Theta(W,C)$ is zero bordant [K]. This implies our statement using the topological h–cobordism Theorem [F].

We will not repeat the definition of $\Theta(W,C)$. Instead we formulate some elementary properties which are enough to show that in our situation $\Theta(W,C)$ is zero bordant. Elements in $\ell_5(\mathbb{Z}/2)$ are represented by pairs $(H(\Lambda^r),U)$, where $H(\Lambda^r)$ is the hyperbolic form on $\Lambda^r \times \Lambda^r$ and $U \subset \Lambda^r \times \Lambda^r$ is a half rank free direct summand. Note that the difference to the ordinary Wall groups is, that there U is an addition self annihilating (a hamiltonian). Note also that we can forget here the quadratic refinement of the form since it is determined by it. Since the ordinary Wall group $L_5(\mathbb{Z}_2)$ vanishes one can characterize zero bordant elements in $\ell_5(\mathbb{Z}_2)$ as follows:

(3) $[H(\Lambda^r),u] \in \ell_5(\mathbb{Z}/2)$ is zero bordant if U has a hamiltonian complement V.

By construction of $\Theta(W,C)$ and some elementary considerations it has the following properties:

(4) If $(H(\Lambda^r),U)$ represents $\Theta(W,C)$ then $(H(\Lambda^r),U^\perp)$ represents $\Theta(W,C')$.

(5) There exists a surjective homomorphism $d : U \twoheadrightarrow H_2(\tilde{C})$ inducing an isometry of the form on U with the intersection form on $H_2(\tilde{C})$.

(6) If $V = \Lambda^s \xrightarrow{f} H_2(\tilde{C})$ is a free Λ–resolution, $\Theta(W,C)$ has a representative $(H(\Lambda^s),V)$ such that d occurring in (5) is equal to f.

Since $H_2(\tilde{C}) = \mathbb{Z}_- \oplus \Lambda^9$ we can take $V = \Lambda^{10}$ with the obvious map $f : V \twoheadrightarrow H_2(\tilde{C})$.

The natural thing for showing that $\Theta(W,C)$ is zero bordant is to prove that in the restriction of $(H(\Lambda^s),V)$ to the ±1–eigenspaces, $V_\pm$ have hamiltonian complements and then to construct from them a hamiltonian complement for V.

The restriction of the hyperbolic form b on $H(\Lambda^8)$ to the ±1 eigenspaces is twice the hyperbolic form on $H(\mathbb{Z}^8)$. In particular the restriction to $V_\pm$ is divisible by two. After dividing by 2 we call this form $b_\pm$ and $V_\pm$ sits isometrically in $H(\mathbb{Z}^8)$.

By assumption (2) the form b_+ vanishes identically on V_+ and thus $(H(\Lambda)_+, V_+)$ represents an element in the ordinary L–group $L_5 = \{0\}$.

We have $V \cong \Lambda^{10} \xrightarrow{f} H_2(\tilde{C}) = \mathbb{Z}_- \oplus \Lambda^9 \longrightarrow H_2(\tilde{C})/\mathrm{rad} = \mathbb{Z}_-^{10}$ and $f|V_-$ maps onto $2\mathbb{Z}_-^{10}$. Thus the form b_- on V_- is $4(E_8 \oplus 2 \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix})/2 = 2 \cdot E_8 \oplus 4 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Since by (4), $(H(\Lambda^{10}), V^\perp)$ represents $\Theta(W,C')$ and the form on $H_2(\tilde{C}')$ is minus the form on $H_2(\tilde{C})$, we know from (5) that the form on $V_-^\perp$ is $-b_-$. Thus we have an isometric embedding $V_- \oplus V_-^\perp = b_- \oplus (-b_-)$ into $H(\mathbb{Z}^{10})$ and we are searching for a hamiltonian complement of V_- in $H(\mathbb{Z}^{10})$.

The different isometry classes of embeddings of a pair of direct summands V_- and $V_-^\perp$ (they are direct summands since V and $V^\perp$ are so) into $H(\mathbb{Z}^{10}) = H$ are equivalently classified by analyzing in how many different ways the hyperbolic form can be reconstructed from the sublattice $V_- \oplus V_-^\perp$. To do this we consider the adjoint $\mathrm{Ad}b_- : V_- \longrightarrow V_-^*$. Denote the cokernel of $\mathrm{Ad}b_-$ by L, a finite abelian group since $\mathrm{Det}\, b_- \neq 0$. On L we have an induced quadratic form $q: L \longrightarrow \mathbb{Q}/\mathbb{Z}$ given by $q([x]) = \frac{1}{2|L|} b_-((\mathrm{Ad}b_-)^{-1}(|L| \cdot x), (\mathrm{Ad}\, b_-)^{-1}(|L| \cdot x))$.

Similarly starting with $V_-^\perp$ we get a quadratic form denoted by $(L^\perp, q^\perp)$. Of course (L,q) and $(L^\perp, -q^\perp)$ are isometric and by means of this isometry identify them with (L,q). We can reconstruct H and the embeddings of V_- and $V_-^\perp$ as follows.

$H = \mathrm{Ker}\,(V_-^* \times (V_-^\perp)^* \longrightarrow L)$, $V_- = \mathrm{Ker}\, p_2 : V_-^* \times (V_-^\perp)^* \longrightarrow (V_-^\perp)^*$,

$V_-^\perp = \text{Ker } p_1 : V_-^* x (V_-^\perp)^* \longrightarrow V_-^*$. Here the map $V_-^* \times (V_-^\perp)^* \longrightarrow L$ is the difference of the projections onto L. This reconstruction follows from a standard argument similar to ([W], p. 285 ff).

Thus we have to analyze the isometries between (L,q) and $(L^\perp, -q^\perp) = (L,q)$ modulo those which can be lifted to isometries of V_-^*. Indeed, (H,V_-) is zero bordant if and only if the corresponding isometry of (L,q) can be lifted to V_-^*. This follows since if V_- has a hamiltonian complement, (H,V_-) is isomorphic to an element which corresponds to Id on L. On the other hand the element corresponding to a liftable isometry of (L,q) has an obvious hamiltonian complement.

Unfortunately there exist isometries of (L,q) which cannot be lifted to V_-^*. We have to show that the corresponding elements of $\ell_5(\mathbb{Z}_2)$ don't occur in our geometric situation. The key for this is that we know that since C and C′ are bordant rel. boundary in $\Omega_4(B,p)$ they are stably diffeomorphic [K], i.e. $C\#r(S^2 \times S^2)$ is diffeomorphic to $C'\#r(S^2 \times S^2)$ for some r and in particular there exists a bordism $\hat{W}$ between $C\#r(S^2 \times S^2)$ and $C'\#r(S^2 \times S^2)$ with $\Theta(\hat{W}, C\#r(S^2 \times S^2))$ zero bordant. Obviously $\hat{W}$ is bordant to $W\#r(S^2 \times D^3)$ $\#r(S^2 \times D^3)$ where the boundary connected sum takes place along C and C′ resp. and W is appropriately chosen. If $(H(\Lambda^2),V)$ represents $\Theta(W,C)$ then $(H(\Lambda^{s+2r}), V \oplus H(\Lambda^r \times \{0\}))$ represents $\Theta(\hat{W}, C\#r(S^2 \times S^2))$. Denote $\hat{V}_- := V_- \oplus H(\Lambda^r \times \{0\})_-$. Then $\hat{L} = L \oplus H(\mathbb{Z}^r)/2$. We know that the isometry of (L,q) corresponding to $\Theta(W,C)_-$ can after adding Id on $H(\mathbb{Z}^r)/2$ be lifted to an isometry of $\hat{V}_-^*$. We call an isometry of (L,q) with this property a <u>restricted</u> isometry.

<u>Lemma:</u> The group of restricted isometries of (L,q) modulo those induced by isometries of V_-^* is trivial.

Before we prove this Lemma we finish our argument that $\Theta(W,C)$ is zero bordant, i.e. V in $H(\Lambda^{10})$ has a hamiltonian complement T. We know that $V_\pm$ have hamiltonian complements $T_\pm$. We also know that V is a direct summand (over Λ) in $H(\Lambda^{10}) = H$. Choose $\mathbb{Z}$–bases a_i of V_+, b_i of V_-, c_i of T_+ and d_i of T_-, such that $(a_i + b_i)/2$ is a Λ–base of V and $a_i \circ c_j = b_i \circ d_j = 2\delta_{ij}$. Then we know that for each d_i there are elements $\alpha_i \in V_+$, $\beta_i \in V_-$ and $\gamma_i \in T_+$ such that $\alpha_i + \beta_i + \gamma_i + d_i = 0 \bmod 2$ in H and $\rho_i := (\alpha_i+\beta_i+\gamma_i+d_i)/2$ form a Λ–basis of H/V. We want to choose these elements so that they generate a hamiltonian, i.e. the form is trivial between those base elements.

Since $a_i + b_i = 0 \bmod 2$ we can assume $\beta_i = 0$. Write $\alpha_i = \Sigma\alpha_{ij}\, a_i$ and $\gamma_i = \Sigma\gamma_{ij} c_j$ with $\alpha_{ij} \in \{0,\pm1\}$ and $\gamma_{ij} \in \{0,1\}$. A simple computation with evaluation of the form implies $\gamma_{ij} = \delta_{ij}$ and thus $\gamma_i = c_i$. Similarly one can show $\alpha_{ij} = \alpha_{ji} \bmod 2$ and $\alpha_{ii} = 0$. Since we are free to change the sign of α_{ij} we can assume $\alpha_{ij} = -\alpha_{ji}$ for $i \neq j$. With these assumptions it is easy to check that $\rho_i \circ \rho_j = 0$ for all i,j and we are finished.

4. Proof of the Lemma. In an equivalent formulation we have to study the following situation. Consider in $H(\mathbb{Z}) \oplus E_8$ the lattice $4 \cdot H(\mathbb{Z}) \oplus 2 \cdot E_8$ and consider $L = H(\mathbb{Z})/4H(\mathbb{Z}) \oplus E_8/2E_8 = L_1 \oplus L_2$ with the induced quadratic form q which is on L_1 given by $q[x] = \frac{1}{8}\, b(x,x)$ and on L_2 by $q[x] = \frac{1}{4}\, b(x,x)$ and $L_1 \perp L_2$. A simple calculation shows that the only isometries of $(L_1, q\,|\,L_1)$ are ±1 and $\pm \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, which obviously can be lifted to $L_1 = H(\mathbb{Z})$. The nontrivial analogous lifting statement holds for L_2 ([BS], p. 416). Thus we are finished if modulo isometries of $H(\mathbb{Z}) \oplus E_8$ each restricted isometry of L preserves L_1 and L_2.

We denote the standard symplectic basis of $H(\mathbb{Z})$ by e and f. Let $g:(L,q) \longrightarrow (L,q)$ be a restricted isometry. Write $g[e] = a[e] + b[f] + [x]$ with $x \in E_8$. Since $g[e]$ has order 4, a or b must be odd. Since g is restricted, $g \oplus \mathrm{Id}$ on $L \oplus H(\mathbb{Z}^r)/2$ can be lifted to an isometry of $H(\mathbb{Z}) \oplus E_8 \oplus H(\mathbb{Z}^r)$ under which e is mapped to $\tilde{a}e + \tilde{b}f + x + 2y + 2z$ where $a = \tilde{a} \bmod 4$, $b = \tilde{b} \bmod 4$, $y \in E_8$ and $z \in H(\mathbb{Z}^r)$. Computing the quadratic form of this element yields $2ab + (x + 2y) \circ (x + 2y) = 0 \bmod 8$.

Since a or b is odd we can after acting with an appropriate liftable isometry assume $a = 1$ or $g[e] = [e] + b[f] + [x]$. Now consider $\hat{g}(e) := e + (b - 4c)f + x + 2y$, where $2b + (x + 2y) \circ (x + 2y) = 8c$. Then $\hat{g}(e) \cdot \hat{g}(e) = 0$. We can extend $\hat{g}$ to an isometry of $H(\mathbb{Z}) \oplus E_8$ by setting $\hat{g}(f) = f$. Then $\hat{g}(e)$ and $\hat{g}(f)$ span a hyperbolic plane in $H(\mathbb{Z}) \oplus E_8$ whose orthogonal complement is isometric to E_8 and we use this isometry to extend $\hat{g}$.

After composing with $\hat{g}^{-1}$ we obtain h with $h[e] = [e]$. Since $h[e] \circ h[f] = \frac{1}{4}$ we must have $h[f] = a[e] + [f] + [y]$. By the same argument as above we obtain an isometry $\hat{h}$ of $H(\mathbb{Z}) \oplus E_8$ with $\hat{h}(e) = e$ and $\hat{h}[f] = a[e] + [f] + [y]$ and after composing again with $\hat{h}^{-1}$ we obtain an isometry which preserves $H(\mathbb{Z})/4H(\mathbb{Z})$ finishing our proof.

5. Some knottings in D^4. Let K be the punctured compact Klein bottle with boundary S^1. We consider smooth embeddings of $(K, \partial K)$ into (D^4, S^3) with fixed relative normal number, $\pi_1(D^4 - K) = \mathbb{Z}_2$, intersection form of the 2–fold ramified covering equal to $<1> \oplus <-1>$ and $(S^3, \partial K)$ a fixed knot. We claim that two such knottings (D^4, K) and (D^4, K') are homeomorphic rel. boundary. The proof is similar as for our Theorem and we indicate the necessary changes.

As in section 2 we choose linear identifications of open tubular neighborhoods of K and K' and denote their complements by C and C'. We identify $\partial C = \partial C' = M$ and choose our identification such that the Spin structures on M agree and the diagram (1) commutes. A similar consideration as in section 2 shows that $H_2(\tilde{C}) = \mathbb{Z}_- \oplus \Lambda$ and the radical of the intersection form is $\mathbb{Z}_+ = H_2(\tilde{C})_+$ and the form on $H_2(\tilde{C})/\mathrm{rad}$ is $2\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Then we proceed as in section 3. Most of the arguments there don't make any special assumptions which are not fulfilled in our situation. The only difference is in the analysis of $(H(\Lambda^2)_{-},V_{-})$. Again this is determined by an isometry of $(L = \text{coker}\ 4\begin{bmatrix}0&1\\1&0\end{bmatrix},q)$. The situation is easier than in section 4, since the lifting problem is simpler. The problem is here whether any isometry on (L,q) is induced from an isometry of $H(\mathbb{Z})$. But as mentioned in section 4 this holds, finishing the argument.

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