Raynaud's theorem on coverings of curves.

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I will in this lecture report on some recent results concerning coverings of algebraic curves in positive characteritic. To a projective smooth and connected algebraic curve X in positive characteristic one associates an invariant which is the p-rank of X, and which is the rank of the pro-p-quotient of the geometric fundamental group of X. The p-rank is a positif integer which is smaller than the genus of X, and X is said to be ordinary if one has equality. One would like to understand how vary this p-rank in étale covers. Let X_η be a "generic" curve in positive characteristic. It is well known that X_η is ordinary and also that all its étale abélien covers are also ordinary. The naturel question to ask is whether every étale covering of the generic curve is ordinary ? The following theorem of Raynaud answers by the negative this question :

Theorem . (Raynaud [Ra]) For any curve X of genus $g \geq 2$ over an algebraically closed field of characteristic $p \geq 0$ there exists an étale connected Galois covering $Y \to X$ of group G which is a solvable group of order prime to p and such that Y is not ordinary.

I also discuss some recent result du to me and F. Pop which concerns the specialisation homomorphism between fundamental groups.

[Ra] M. Raynaud. Revêtements étales de courbes algébriques en caractéristique positive et ordinarité, preprint.