

COVERINGS OF P-ADIC PERIOD DOMAINS

GERD FALTINGS

Classically A.Grothendieck conjectured a correspondence between étale and crystalline cohomology, the "mysterious functor". J.M.Fontaine has defined such a functor, relating certain Galois-representations of p -adic fields (crystalline representations) to filtered vectorspaces with Frobenius-actions (admissible objects). This gives an equivalence of categories between certain Galois-representations, called crystalline, and certain crystalline tuples which are called admissible. It has been shown that for p -adic field of finite ramification-index admissible is equivalent to a certain stability condition, named weakly admissible.

The parameter spaces for weakly admissible filtration are rigid spaces, the classical examples being the Drinfeld halfspace or the Lubin-Tate space. It was conjectured that the associated Galois-representations glue to an étale p -adic sheaf.

For infinite ramification U.Hartl has shown that this need not be true, as for fields of infinite ramification weakly admissible does not imply admissible. He remarked that the admissible points form an open Berkovich subspace. The subject of this talk is to demonstrate that the desired étale sheaves exist on this Berkovich space.

For the moment this is done only for period domains classifying p -divisible groupes of constant slopes. However I see in principle no obstruction to extending the method to other cases. Note that as in the classical complex case one sometimes encounters the difficulty that because of Griffiths transversality there are no more universal families. Over the flag variety which parametrises the Hodge filtrations the universal family does not satisfy it.

The method of proof uses a generalisation of Fontaine's ring A_{cris} to higher dimensional bases. One uses that admissibility is equivalent to the possibility to solve certain equations in these rings. If one has such a solution at a point it extends to an approximate solution in a neighbourhood, which then is corrected to an exact solution by an iteration procedure.

Once one has the étale local systems one can show (using work of Breuil and Kisin) that in our case they are actually Tate-modules of p -divisible groups.

MAX-PLANCK-INSTITUT FÜR MATHEMATIK, VIVATSGASSE 7, 53111 BONN, GERMANY