A stochastic approach to the Euler-Poincaré number of the loop space of a developable orbifold

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OF A DEVELOPABLE ORBIFOLD

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Introduction.

In the physics of string theory, one consider the string propagation on a manifold M quotient by a finite group of symmetries G . When the group action is non-free, the quotient space M/G is in general not a smooth manifold, but one with singularities, a so called developable orbifold. In the discussion of string vacua for M/G, one has to consider the configuration of the closed (parametrized) loops of M together with all the loops twisted by elements of G. By consideration of modular invariance of the theory, Dixon, Harvey, Vafa and Witten [D.H.V.W] introduced the following 'orbifold Euler characteristic ' of the quotient of M by G as the appropriate Euler number for the purpose of string theory :

$$(0.1) \quad \chi(M,G) = 1/|G| \sum \chi(M^{<g,h>})$$

where the summation runs over all commuting pair in GxG, and $M^{<g,h>}$ denote the common fixed point set of g and h. For a free action it is the well known fact : $\chi(M,G) = \chi(M/G)$. The connection of this expression with the representation theory of the group G leads to the identification of $\chi(M,G)$ with the Euler characteristic of equivariant K-theory $K_G(M)$ which was noted by [A.S]. However, the string calculation of Euler number is expected to agree with the Euler number of a "correct" resolution $M/G°$ of rhe singular space M/G, at least for manifolds with SU(n)-holonomy. For $\dim_C M = 2$, [H-H] showed that the equality

$$(0.2) \quad \chi(M,G) = \chi(M/G°)$$

holds for $M/G°$ the minimal resolution of M/G. When $\dim_C M = 3$ and G abelian, $\chi(M,G)$ is also identified with $\chi(M/G°)$ for $M/G°$ being the "minimal" toroidal resolution of M/G constructed by the methods in toric geomtry by [R-Y], [R], and also by [M,O,Pe]. It seems that this phenomenon should hold for a general reasonable class. Even though the formula of orbifold Euler characteristic was obtained by stringists using the physicist's ideas which is quite natural, it is in some sense unsatisfactory because a clearer mathematical nature of "strings" still rely on a rigorous mathematical description of the intuition behind it. Here we propose a mathematical treatment using probabilistic method based on Malliavin calculus, which can justify some intuitive and heuristic method of the physical arguments. The formulation might shed some light of the "string" nature of toric geometry which has been a useful device in the study of string compactification.

In order to give an interpretation to these ideas, we need to consider an element of volume over the twisted loop space, and unfortunately we meet the problem that there is no riemannian measure over the loop space of an orbifold. The idea is to use the twisted Bismut measure, which extends in the case of twisted loop space the measure which was introduced in [Bi_5] in order to explain the relation between the cohomology of the loop space and the index theory (See [H.K] in the flat case). This measure in the case of non twisted loop is used in [$J.L_1$] in order to do a L^p theory of Chen forms. But no differential operation is given in [$J.L_1$].

Such differential operations are known since a long time in the Malliavin Calculus for the Wiener measure : these are the Malliavin derivatives ([Gr_1]) and the Ornstein-Uhlenbeck operator ([M]). [Sh] and [Ar-M] study differential forms over the Wiener space and the exterior derivative. [Ar_1],[Ar_2] do an extensive study of the index of the the Dirac operator over the flat loop space in the case of the free field or with interacting terms : they give a path integral representation of the index of such operators over the loop over the loop, strongly inspired by the work in the scalar case of Hoegh-Krohn ([H.K]) (See [$J.L.W_1$] and [$J.LW_2$] for physicist references).

For the analysis in infinite dimensional curved spaces, there exist different types of theory (We can refer to [Ma] for a survey).

-The analysis over infinite dimensional manifold, which was used by the Russian shool. The manifold's structure is very important in such cases ([B.S],[D.F])

-The quasi-sure analysis ([Ge],[$A.M_1$]) which works over finite codimensional manifold of the Wiener space (See [K] for forms).

-The analysis over loop groups ([$A\text{-}M_2$],[Gr_2],[Gr_3]) which is closer to the purpose of this paper, but with a different tangent space, which uses deeply the stucture of the group. There is for the moment no manifold strcture in this theory as well in the next theory.

The present paper is more related to [Le_4] and [Le_5], where the case of the free loop space of the Riemannian manifold is considered. Some connections are introduced over the free loop space, integration by parts are done, which allows us to define Malliavin's derivatives of every order and to define an Ornstein-Uhlenbeck operator invariant by rotation. Moreover it is proved that the Bosonic part of the Witten current in [Le_5] is a smooth measure (See [Ge-J-P] for the algebraic definition of the Witten current). A relation between the exterior stochastic derivative and the Hochschild homology of forms of the manifold is given, which extends this well known fact in the case of smooth loop to the case of stochastic loop. Moreover, the circle action is diagonalised in preparation of [$J.L_2$].

[$J.L_2$] defined a non equivariant regularised exterior derivative over the full space of forms of the loop space. Its adjoint is computed. A rigorous conjecture for the index of the regularised de Rham operator is given. By localisation, it is the Euler-Poincaré number of the manifold. The situation becomes more complicated for the case of the equivariant Dirac operator over the loop space with the relationwith the Witten genus and for the case of the equivariant signature operator over the loop space with the relation with the elliptic genus : some topological obstructions are met ([Be],[Se],[Wi]) and in fact in [J.L$_2$] there is an extension of the Taubes construction of the Dirac operator over infinitisimaly loop ([T]) only over a small neighborhood, by using the stochastic calculus. In order to define the "restriction" to these non scalar operators to infinitisimaly small loop (that means over the family of Brownian bridges over the set of tangent spaces), the Bismut mesure in small time is introduced and some limit theorem are perfomed, which correspond to the high temperature limit in the stochastic context and which are of the domain of the computations done in [Bi$_3$],[I.W] ,[Hs] and [Le$_2$].

The purpose of this paper is to do analoguous computations for the regularised exterior derivative for twisted loop spaces : the loop space of a develop able orbifold appears namely as an orbifold of twisted loops. A scalar calculus over each sector of twisted loop is done. A diffusion process is constructed, and some rough localisation is perfomed for the diffusion process (See [A.L.R] for non twisted loops). The big difference with [J.L$_2$] is that the limit model is related to the computation of [Bi$_4$] instead of the model of [Bi$_3$], because the twisted loop concentrate over the loop of the fixed point of an element of G in small time.

The introducing reference for stochastic geometry can be [El],[Em],[Le$_3$] and [R.W].

Both of the authors like to thank Prof.Dr.F.Hirzebruch for his warm hospitality in the Max-Planck-Institut where this work was done. The first author also thanks the Von Humboldt Foundation for financial support.

I SCALAR CALCULUS OVER TWISTED LOOP SPACES.

I 1) Integration by parts for distinguished vector fields.

Let M be a compact Riemannian manifold and let G be a finite group acting over M. By averaging, we can suppose that G is a group of isometries. Let L be the Laplace-Beltrami operator over M and $p_t(x,y)$ the associated heat kernel. $P_{1,x,y}$ is the law of the Brownian bridge starting from x and going to y in time 1. Let H_g be the space of twisted loop going from any x and arriving in gx in time 1. Let μ_g the measure over H_g

(1.1) $d\mu_g = p_1(x,gx)\, P_{1,x,gx}\, dx / \int_M p_1(x,gx)\, dx$

We denote the space of L^2 function associated by H_g. Let X_g the vector field :

(1.2) $\tau_t(X_0(\gamma(0)) + \int_{[0,t]} h(s)\, ds - t\, X_0(\gamma(0)) + t\, \tau_1^{-1}\, dg\, X_0(\gamma(0)) = \tau_t\, H(t)$

$h(s)$ is equals to $\sum h_i(s)\, X_i(\gamma(0))$ where each h_i is deterministic such that $\int_{[0,1]} h_i(s)\, ds = 0$. These vector fields play the role of the distinguished vector fields given in [Le_4] and in [Le_5] . But the boundary conditions are now $X(1) = dgX(0)$ because we look at twisted loops. Let F be a smooth cylindrical functional $F(\gamma(t(1)),..,\gamma(t(r)))$. We have :

THEOREM I.1:

$$\mu_g[<dF,X_g>] = \mu_g[\, F \operatorname{div} X_g] \tag{1.3}$$

where

$$<dF,X_g> = \sum < d_{\gamma(t(i))}F(\gamma(t(1)),....,\gamma(t(r))),X_g(t(i))> \tag{1.4}$$

and where

$$\operatorname{div} X_g = \operatorname{div} X_g(\gamma(0)) + \int_{[0,1]} <\tau_s H'(s),\delta\gamma(s)> - 1/2 \int_{[0,1]} < S_{X_g(s)},\delta\gamma(s)> - \int_{[0,1]} \operatorname{Tr} s\, \tau_s^{-1} R(d\gamma(s), \tau_s.)\, \tau_s \tau_1^{-1}\, dg\, X_g(\gamma(0)) \tag{1.5}$$

where S *is the Ricci tensor over* M *and* R *the curvature tensor.*

Remark : Let us remark that the last term in the divergence can be computed by means of the Ricci tensor and is equals to zero when the manifold is Ricci flat.

Proof : The proof is very similar to the proof of [Le_4]. We begin to work over the path space, that means the space of applications from [0,1] into M endowed with the path space measure $dx\, P_1^x$ where P_1^x is the law of the Brownian motion starting from x and arriving at time 1 in x. Let ϕ a smooth function over MxM with a small support over the diagonal equals to 1 over a small neighborhood of the diagonal such that $g\, \gamma(0)$ and $\gamma(1)$ are joined by a unique geodesic if $\phi(g\gamma(0),\gamma(1))$ is not equal to 0. Let us denote by $\tau(\gamma(1),g\gamma(0))$ the parrallel transport from $g(\gamma(0))$ to $\gamma(1)$ along this geodesic. We begin by enlarging the vector field X_g over the twisted loop space into a vector field $X_{1,g}$ over the path space by putting :

$$X_{1,g}(t) = \phi(g\gamma(0),\gamma(1))\Big(\tau_t\big(X_0(\gamma(0)) + \int_{[0,t]} h(s)\, ds - tX_0(\gamma(0) + t\tau_1^{-1}\, \tau(\gamma(1),g\gamma(0))\, dg\, X_0(\gamma(0))\big)\Big) \tag{1.6}$$

Let N be a subdivision of [0,1], and let $X_{1,g}{}^N$ be the associated vector field and let consider the polygonal approximation of γ : this polygonal approximation of γ works only if $\gamma(t(i))$ and $\gamma(t(i+1))$ are close, but the contribution of the path where $\gamma(t(i))$ and $\gamma(t(i+1))$ are far goes to 0 when N goes to the infinity, as it is explained in [Le$_4$],[Le$_5$]. We know by integrating by parts in finite dimension that

$$(1.7)\quad \nu[<dF,X_{1,g}{}^N>] = \nu[F \operatorname{div} X_{1,g}{}^N]$$

Moreover, by using the Malliavin Calculus, we know that for all x, y that

$$(1.8)\quad E_{1,x,y}[<dF,X_{1,g}{}^N>] \to E_{1,x,y}[<dF,X_{1,g}>]$$

and that

$$(1.9)\quad E_{1,x,y}[F \operatorname{div} X_{1,g}{}^N] \to E_{1,x,y}[F \operatorname{div} X_{1,g}]$$

where the divergence is computed from (1.5) by taking the derivative more of $\phi(\tau(\gamma(1),g\gamma(0))$ because we are integrating by parts over the path space. By using the fact that g is an isometry, and the relation $X_{1,g}{}^N(1) = dg\ X_{1,g}{}^N(0)$, we arrive at the same cancellation at the end when N goes to infinity as the cancellations enregistred for non twisted loop. The only difference is that we don't need to take derivative of $\tau(\gamma(1),g\gamma(0))$ in the approximation limit procedure, so therefore the theorem, by considering the matrix from $T_{\gamma(0)}$ into (by $T_{\gamma(0)}$) $\tau_1{}^{-1}$ dg instead of $\tau_1{}^{-1}$ in the case of non twisted loop. But dg ($\gamma(0)$) has a derivative equals to 0 over a vector field $\tau_t\, t\, X$: this explains the fact that no derivative of dg appears in the last counterterm.

I.2. Dirichlet form and Ornstein-Uhlenbeck operator..

The tangent space is the space of vectors $X(s) = \tau_s H(s)$ with H(s) with finite variations such that $X_1 = dg\ X_0$. As Hilbert structure, it should be possible to choose the Hilbert structure $\|X(0)\|^2 + \int_{[0,1]} <DX(s),DX(s)> ds$ where $DX(s) = \tau_s\ H'(s)$ is the covariant derivative over the loop. But we will split our tangent space T_γ into $\Sigma_{\mathbb{Z}}\ T_\gamma{}^n$ in an orthogonal sum with a different metric in order to simplify the computations.

If $n > 0$, $T_\gamma{}^n = \{\tau_t\, 2^{1/2} \int_{[0,t]} \cos(ns)\, ds\, e = X(n,e)(t)\ \}$.

If $n < 0$, $T_\gamma^n = \{ \tau_t 2^{1/2} \int_{[0,t]} \sin(ns)\, ds\, e = X(n,e)(t) \}$.

If $n = 0$, $T_\gamma^0 = \{ \tau_t (e - te + t\tau_1^{-1} dg\, e) = X(0,e)(t) \}$

The Hilbert strucure over each piece T_γ^n of T_γ is given by $\|e\|_{\gamma(0)}^2$.

There is a connection which preserves the metric. This arises from the Levi-Civita connection Γ over the manifold :

(1.10) $X(n,\Gamma e)(t) = \Gamma(X(n,e)(t))$

This connection preserves by definition the splitting of T_γ into T_γ^n.

Let us introduce positive numbers $A(n)$ such $A(n) < C|n|^p$ $2p< 1$. Let E' the following Dirichlet form :

(1.11) $E'(F,F) = \Sigma_{n,i,g}\, \mu_g[A^2(n) <dF,X(n,e(i))>^2]$

where $X(n,e(i))$ is a basis of T_γ^n.

LEMMA I.2: E' *is closed, defined over a dense set which separates the twisted loop and tight..*
Proof : E' is densely defined. We choose as core Λ the set of cylindrical functions $F(\gamma(t(1)),..,\gamma(t(r)))$. Over T_γ^n $n\neq0$, we choose as orthonormal basis the natural orthonormal basis which comes from $T_{\gamma(0)}$. We have :

(1.12) $|<dF,X(n,e(i))>| \leq C/(|n| + 1)$

Therefore :

(1.13) $\Sigma A(n)^2 <dF,X(n,e(i))>^2 \leq \Sigma A(n)^2/(|n|+1)^2 \leq C$

with C deteministic. Therefore, E' is densely defined over a set of functions which separate the loop.

-)E' is closed. Let us suppose that $F_p \to 0$ for F_p belonging to the core and that :

(1.14) $\mu_g[\Sigma A(n)^2 (<dF_p,X(n,e(i))> - G(n))^2] \to 0$

when $p \to \infty$. Then $G_n = 0$.

Namely for all cylindrical functional F,

$$(1.15)\quad \mu_g[<dF_p,X(n,e(i))> F] = \mu_g[F_p \operatorname{div} X_n F] - \mu_g[F_p <dF,X(n,e(i))>]$$

which tends to 0. Therefore $<dF_p,X(n,e(i))>$ tends to 0 in $L^2(\mu_g)$ and therefore $G_n = o$ (We used local sections of smooth orthonormal basis of $T_{\gamma(0)}M$).

-E' is tighted for the uniform convergence. Let F(x,y) a smooth ≥ 0 function over MxM such $F(x,y) = d^2(x,y)$ if x and y are closed. Let $G(\gamma)$ the random variable :

$$(1.16)\quad G(\gamma) = \int_{[0,1]}\int_{[0,1]} F(\gamma(s),\gamma(t))^{\rho}{}_{/|t-s|^{\alpha}} ds\, dt$$

which is finite if $\rho > \alpha$. Let us compute $<dG(\gamma), X(n,e(i))>$. It is enough to take $n \neq 0$.

$$(1.17)\quad \begin{aligned} |<dG(\gamma),X(n,e(i))>| &< |\int_{[0,1]}\int_{[0,1]} F(\gamma(s),\gamma(t))^{\rho-1}/|t-s|^{\alpha}\, |< d_{\gamma(s)}F(\gamma(s),\gamma(t)),X(n,e(i))(s)> \\ &+ < d_{\gamma(t)}F(\gamma(s),\gamma(t)),X(n,e(i))(t) > |ds\, dt < C/|n| \int_{[0,1]}\int_{[0,1]} F(\gamma(s),\gamma(t))^{\rho-1}{}_{/|t-s|^{\alpha}}\, ds\, dt \end{aligned}$$

Therefore if $\rho - 1 > \alpha$, $\int_{[0,1]}\int_{[0,1]} F(\gamma(s),\gamma(t))^{\rho-1}{}_{/|t-s|^{\alpha}}\, ds\, dt$ is finite. Moreover, if we put $\alpha = 1 + 2\beta\rho$, $G < C$ is compact if $\beta < 1/2$ for the uniform norm (See [A.V] for the case of Wiener submanifold).The following theorem can be deduced classically from the previous lemma.

THEOREM I.4 :*To the Dirichlet form is associated outside a set of capacity 0 a process* $w_t(\gamma)$ *for* μ_g.

Let us consider the operator L_A associated to the Dirichlet form. It has the definition :

$$(1.18)\quad L_A F = -\sum A(n)^2 <d<dF,X(n,e(i))>,X(n,e(i))> + \sum A(n)^2 <dF,X(n,e(i))> \operatorname{div}X(n,e(i))$$

THEOREM I.5 : L_A *is defined over the core* $\AA$ *if* $4\rho < 1$.

Proof: Only the case $n \neq 0$ is important.

$$(1.19)\quad |<d<dF,X(n,e(i))>,X(n,e(i))>| < C(n) /(n^2+1)$$

and the sequence of random variable C(n) is bounded in L^2, this from the relation :

(1.20) $\Gamma_X \tau_t = \tau_t \int_{[0,t]} \tau_s^{-1} R(d\gamma(s), X(s))\, \tau_s$

for the Levi-Civita connection Γ. So only the second part in the definition of $L_A F$ put a problem. Let us consider only the $n > 0$ part :

$$\mathrm{div}X(n,e(i)) = \int_{[0,1]} < \tau_s \cos(ns)e(i), \delta\gamma(s) > + 1/2 \int_{[0,1]} < S_{X(n,e(i))(s)}, \delta\gamma(s) > \quad (1.21)$$
$$+\text{counterterms}$$

The counterterms have a behaviour in $C(n)/n$ with $C(n)$ uniformly bounded in $L^2(\mu_g)$ and do not put any problem.

Let us consider the j^{th} part of the derivative of $F(\gamma(t(1)),..,\gamma(t(r)))$. Let us consider the element of $L^2[0,1]$ whose Fourier series is 0 if $n < 0$ and $(A(n)^2/n)\,(\sin(nt(j)) - 1)$. Denote it by $h_{t(j)}(s)$. (The convergence is performed because $4\rho < 1$). In the first contribution of the divergence in the operator, we recognise :

(1.22) $\int_{[0,1]} < \tau_s < d_{\gamma(t(j))} F(\gamma(t(1)),\ldots,\gamma(t(r))), \tau_{t(j)} h_{t(j)}(s)\, e(i) >, \delta\gamma(s) >$

which belongs in $L^2(\mu_g)$ because we recognise a non-anticipative Itô integral.

LEMMA I.7 : *Let F be a cylindrical functional.*

(1.23) $\mu_g[\, \exp[\, C|L_A F|\,]\,] < \infty$

for all C if $4\rho < 1$.

Proof : The part in $L_A F$ which comes from (1.22) satisfies clearly (1.23). The part in $L_A F$ which comes from the first sum in (1.18) satisfies easily (1.22). Namely only the derivative of the parrallel transport put any difficulties, but it is overcomed by (1.19) and by recognising after a non anticipative Itô integral as in (1.22). It remains to treat the contribution of the counterterms in (1.21). Let us study for instance the contribution of

(I.24) $\quad \sum_{n>0,i} A(n)^2 < dF, X(n,e(i)) > \int_{[0,1]} < S_{X(n,e(i))(s)}, \delta\gamma(s) > \; = \sum_j \int_{[0,1]} < S_{Y(j)(s)}, \delta\gamma(s) >$

where $Y(j)$ is a process of the same type of (1.22). This non anticipative integral is in particular exponentialy integrable. The same holds for the last counterterm.

I.3.Localisation.

We can handle now with the following theorem which could justify that the equivariant Euler number under the geometrical action of h should be localised over the twisted loop in g of the fixed point of h.

THEOREM I 8:

(1.25) $\mu_g[d(w_t(\gamma),\gamma) > \delta] < \exp[-C/t]$

when t is tending to 0.

Proof: d is the uniform distance. Let us cut the time interval in t^{-1} time interval $[s(i),s(i+1)]$ of the same length. The event $d(w_t(\gamma),\gamma) > \delta$ is included in the union of the events $\{ d(w_t(\gamma)((s(i)),\gamma(s(i)) > \delta \} = O_i$ and of the events $\{ \mathrm{Sup}_{[s(i),s(i+1)[} d(w_t(\gamma)(s),w_t(\gamma)(s(i)) > \delta'' \} = O'_i$. By the stationarity of the process :

(1.26) $\exp[-C/t] \geq \mu_g \{ \mathrm{Sup}_{[s(i),s(i+1)[} d(\gamma(s),\gamma(s(i))) > \delta''\} \geq \mu_g\{O'_i\}$

Since the number of O'_i is controlled by t^{-1}, the second term is controlled by $\exp[-C/t]$ when $t \to 0$. Let us estimate $\mu_g\{O_i\}$. By rescovering M by a set of small balls, O_i can be included in a finite set of $O_{i,j}$ such that :

(1.27) $O_{i,j} = \{ | g_j(w_t(\gamma)(s(i)) - g_j(\gamma(s(i))) | > \delta''\}$

The g_j are independent of the s_i and δ'' too. Since $g_j(\gamma(s(i))$ belongs in the domain of L_A, quasi-surely, we have :

(1.28) $g_j(w_t(\gamma(s(i))) - g_j(\gamma(s(i))) = M_t + \int_{[0,t]} (L_A g_j)(w_s(\gamma))\, ds$

M_t is a martingale whose the derivative of the right bracket is smaller than C because we take a coordinate function. Therefore :

(1.29) $\mu_g[|M_t| > C] < \exp[-C/t]$

Moreover by Jensen-inequality,

(1.30) $\mu_g[\exp[|\int_{[0,t]} |L_A g_j|(w_s(\gamma))\, ds\, /t|]] < C\, \mu_g[\int_{[0,t]} \exp[| L_A g_j|(w_s(\gamma))]\, ds/t|$

for the stationarity of $w_s(\gamma)$. We deduce from this that

(1.31) $\mu_g[|\int_{[0,t]} |L_A g_j(w_s(\gamma))|\, ds > C] < \exp[-C/t]$

Therefore the result.

II REGULARISED DIXON-HARVEY-VAFA-WITTEN EULER'S NUMBER FROM THE LOOP SPACE OF A DEVELOPABLE ORBIFOLD.

II 1) Regularised de Rham operator over the twisted loop space.

Let γ a twisted loop in H_g, and let T_γ its tangent space with the Hilbertian structure of the part I. $T_\gamma = \sum T_\gamma^n$, the sum being taken over the relative integers.Let ΛT_γ^n the exterior algebra associated to $T_\gamma$with the structure coming from each T_γ^n. The connection Γ pass to ΛT_γ. Let $\mathring{A}_g$ the set of sections of the shape $\sigma = \sum F_I(\gamma(t(0),\gamma(t(0)),\ldots,\gamma(t(r)))\, X(I)(\gamma)$ for a finite sum where F_I is a cylindrical functional and where $X_I = X(n_1)(e_1)\wedge\ldots\ldots\wedge X(n_r)(e_r)$. Let us remark that $\Lambda(T\gamma)$ is canonically isomorphic to $\Lambda(\gamma_0) \wedge \Lambda(\gamma_0,H)$ where $\Lambda(\gamma_0,H)$ is the Fermionic Fock space associated to the L^2 Hilbert structure endowed with the flat Brownian bridge in the tangent space of the starting point. Modulo this isomorphism, we take random section which have only a finite number of components which are not equals to zero in this bundle over M in order to define $\mathring{A}_g$ and the coordinates are cylindrical functionals. For σ belonging to $\mathring{A}_g$, we define if $e(i)$ is a local section of orthonormal basis of $T_{\gamma(0)}M$, $d_{r,g}$ by

$$d_{r,g}\sigma = \sum_I \{ \sum_{(n,i)} A(n) <dF_I(\gamma(t(0)),\ldots,\gamma(t(r))), X(n,e(i))(\gamma)> X(n,e(i))(\gamma) \wedge X(I)(\gamma) + \tag{2.1}$$

$$\sum_i A(0)\, F_I(\gamma(t(0),\ldots,\gamma(t(r))\, X(0,e(i)) \wedge \Gamma_{X(0,e(i))(\gamma)} X(I)(\gamma)\}$$

The first sum is taken over the finite number I of component F_I of σ in the distinguished basis

$X(I)(\gamma)$ of the exterior algebra ΛT_γ and the second is involved with the derivatives along the distinguished vector fields of the form. Since the connection Γ is a connection which preserves the metric over ΛT_γ, we can write (2.1) more concisely :

$$(2.2)\quad d_{r,g}\sigma = \Sigma_i A(n)\, X(n,e(i)) \wedge \Gamma_{X(n,e(i))}\sigma.$$

The operator does not depend from the choice of the local smooth section of orthonormal basis e(i) we choose. In a particular case, it can be useful to choose a normal system of coordinate in order to determine the operator. We can compute $d^*_{r,g}$ over $\mathring{A}_g$. Namely,

$$(2.3)\quad \mu_g[\langle d\langle\sigma,\sigma'\rangle,X\rangle] = \mu_g[\langle\sigma,\sigma'\rangle \operatorname{div} X] = \mu_g[\langle\Gamma_X\sigma,\sigma'\rangle + \langle\sigma,\Gamma_X\sigma'\rangle]$$

Therefore

$$(2.4)\quad \Gamma^*_X\sigma = -\Gamma_X\sigma + \sigma \operatorname{div} X.$$

This allows to show that :

$$(2.5)\quad d^*_{r,g} = -\Sigma A(n)\, \Gamma_{X(n,e(i))}\, i_{X(n,e(i))}\sigma + \Sigma\, i_{X(n,e(i))}\sigma \operatorname{div} X(n,e(i))$$

Let us recall (See [J.L$_2$]) that the sum in (2.1) is infinite but converges becaus $2\rho < 1$ and that in $d^*_{r,g}\sigma$ the sum is finite. $d_{r,g} + d^*_{r,g}$ and is a symmetric operator therefore closable and $d^*_{r,g}$ are closable too.

Let us show that $(d_{r,g} + d^*_{r,g})^2$ is defined over $\mathring{A}_g$. For this we have to suppose $4\rho < 1$. We have :

$$d_{r,g}\, d^*_{r,g}\, \sigma = \Sigma_{(n,i)} A(n)\, X(n,e(i)) \wedge \Gamma_{X(n,e(i))}\{ -\Sigma_{(m,j)} A(m)\, \Gamma_{X(m,e(j))} i_{X(m,e(j))}\sigma +$$

(2.6)

$$\Sigma_{(m,j)} A(m)\, i_{X(m,e(j))}\, \sigma \operatorname{div} X(m,e(j))\}$$

The sum in {} is in fact finite. We have only to show that if we take $\langle dF_I, X(n,e(i))\rangle$, we can reach this from the core of cylindrical functionals because the parrallel transport appears in such expressions. This comes from the Bismut's formula :

(2.7) $\Gamma_X \tau_t = \tau_t \int_{[0,1]} \tau_s^{-1} R(d\gamma_s, X_s) \tau_s$

($[Bi_2]$, $[Le_4]$,$[Le_5]$) and from the fact that $2\rho < 1$.

Let us now study the behaviour of $d^*_{r,g} d_{r,g} \sigma$. It equals to

$$\sum A(n) \operatorname{div} X(n,e(i)) i_{X(n,e(i))}\{ \sum A(m) X(m,e(j)) \wedge \Gamma_{X(m,e(j))} \sigma\} - \sum A(n) \Gamma_{X(n,e(i))}$$

(2.8)

$$i_{X(n,e(i))}\{\sum A(m) X(m,e(j)) \wedge \Gamma_{X(m,e(j))}\sigma\}$$

The sum is finite, except for the most embarassing term which is

(2.9) $\sum A(n)^2 \operatorname{div} X(n,e(i)) \Gamma_{X(n,e(i))}\sigma - \sum A(n)^2 \Gamma_{X(n,e(i))} \Gamma_{X(n,e(i))} \sigma$

But if we work in a local chart, we can compare the problem of the convergence of this series to the problem of the convergence of L_A and show it is converging in $L^2(\mu_g)$ as in the first part since $4\rho < 1$.

The sum in $d^*_{r,g} d^*_{r,g} \sigma$ is finite and does not put any problem of convergence.

The sum in $d_{r,g} d_{r,g} \sigma$ is infinite but its convergence comes from the fact that

(2.10) $\mu_g[\,|<d<dF_I, X(n,e(i))>, X(m,e(j))>|^2] \leq C/(n^2+1)(m^2+1)$

using (2.7).

Remark : Let ω be the form over the twisted loop space which to a vector associates $<\omega(\gamma(s)),X_s>$. It is the reciproque image of the one form ω in M by the evaluation map which associates to a twist loop its value in time s. It belongs to the domain of $d_{r,g}$ and $d^*_{r,g}$ For this, we expand X_s in the basis given by T_γ^n and we see that this form is the series $\sum <\omega(\gamma(s)),X(n,e(i))(s)> X(n,e(i))$. There is the parrallel transport which appears in $<\omega(\gamma(s)),X(n,e(i))(s)>$ but can be handled by the formula (2.7) which allows to show that this form belongs to the domain of $d_{r,g}$ and of $d^*_{r,g}$ because $2\rho < 1$.

The Laplacian $(d_{r,g}+ d^*_{r,g})^2 = \Delta_{r,g}$ is densely defined and symmetric, therefore closable.

Let us introduce the geometrical action h : to a twisted loop $\gamma(s)$ it associates the twisted loop $h\gamma(s)$. It is an isometry from H_g into $H_{hgh^{-1}}$ which preserves the splitting of T_γ into $\sum T_\gamma^n$. This comes from the fact that h is an isometry, and if γ is the Brownian bridge between x and gx, $h\gamma$ is the bridge between hx and $hgx = (hgh^{-1})hx$. Moreover the parrallel transport between $h\gamma(0)$ and $h\gamma(t)$ is nothing else than $dh\, \tau_t (dh)^{-1}$ since h is an isometry. Therefore the isometry between T_γ^n and $T_{h\gamma}^n$ is given by $e \rightarrow dhe$. The most difficult part to see this is for a vector field of the type $\tau_t(e - te + t\, \tau_1^{-1}\, dg\, e)$. It is transformed in a vector of the type

$$\tau_t(h\gamma)(dhe - t\, dhe + t\, dh\, \tau_1^{-1}\, dge) = \tau_t(h\gamma)(dhe - tdhe + tdh\, \tau_1^{-1}(dh)^{-1}dh\, dg\, (dh)^{-1}\, dh\, e)$$

(2.11)

$$= \tau_t(h\gamma)(dh\, e - t\, dh\, e + t\, (dh\, \tau_1\, dh^{-1})^{-1}\, d(hgh^{-1})\, dh\, e)$$

The conclusion follows from $dh\, \tau_1\, dh^{-1} = \tau_1(h\gamma)$.

Moreover, h lifts to an application from $\AA_g$ to $\AA_{hgh^{-1}}$ which is an isometry for the natural L^2 structure over these two spaces. Since h preserves the splitting of T_γ into the sum of T_γ^n, *since the A(n) are independent of the choosed starting point*, and since h preserves the Levi-Civita connection over TM, we deduce the following equalities of operators with their domain :

$$h\, d_{r,g} = d_{r,hgh^{-1}}.h$$

$$h\, d^*_{r,g} = d^*_{r,hgh^{-1}} . h$$

(2.12)

$$h\, (d_{r,g} + d^*_{r,g}) = (d_{r,hgh^{-1}} + d^*_{r,hgh^{-1}})\, h$$

$$h\, \Delta_{r,g} = \Delta_{r,hgh^{-1}}\, h$$

The Hilbert space of loop space of a developable orbifold can be identified with the quotient of the union of the sectors H_g by the geometrical action of G over the union of H_g. Therefore, formally, the Euler-Poincaré characteristic of this orbifold of twisted loop space is formally given by $1/|G| \sum Tr_s(\exp[-t\Delta]h)$, the sum being taken over the elements of the group and the expression

Tr_s being the difference of the trace over positive forms and of the over odd forms ([H.Z]). This quantity is formaly equals too to $1/|G| \sum Ind_h(d_r+d_r^*)$.

But Δ preserves each femionic sector, and so only the contribution of the sectors which are kept by the geometrical action of h need to be taken in the equivariant index (It is the diagonal contribution of h). A sector is kept by h if gh = hg. So only sum over commuting pairs (g,h) have to be taken in the Hirzebruch's formula.

We can handle now with the following conjecture:

Conjecture: If $A(n) > |n|^p$

-) $d_{r,g} + d^*_{r,g}$ has a self-adjoint extension.

-)If g and h commute, $\chi(M^g \cap M^h) = Ind_h(d_{r,g} + d^*_{r,g})$.

-)tr $\exp[-t\,\Delta_{r,g}]h$ is finite and $Ind_h(d_{r,g}+ d^*_{r,g}) = Tr_s \exp[-t\Delta_{r,g}]h$

This conjecture could show that the regularised Euler number of the loop space of a developpable compact orbifold is given by the Dixon-Harvey-Vafa-Witten (0.1)formula given in the introduction.

II2) An heuristic proof of the conjecture.

Over H_g, instead of putting the measure $(1/\int_M p_1(x,gx)\,dx)$. $p_1(x,gx)\, P_{1,x,gx}\, dx = \mu_{1,g}$, we choose the measure in small time $(1/\int_M p_{\varepsilon^2}(x,gx)\,dx)\ p_{\varepsilon^2}(x,gx)\, P_{\varepsilon^2,x,gx}\, dx = \mu_{\varepsilon,g}$. This measure concentrates when ε is small over the fixed point M^g of g because $p_{\varepsilon^2}(x,gx) \le \exp[-C/\varepsilon^2]$ when $x \neq gx$ (See [Bi$_4$]). As in [J.L$_2$], we divide the metric in T_γ^n, $n\neq 0$, by ε^{-2} such that an original orthonormal basis is multiplied by ε, although it is kept as a form (See [Bi$_7$] and [Le$_3$]). The contribution of T_γ^0 is more complicated to handle, because there is two parts in T_γ^0 : the part which is transversed to the fixed point set and the part which is tangent to the fixed point set. Of course this distinction works only if $\gamma(0)$ is closed to the fixed point set. If $\gamma(0)$ is close to the fixed point set, we can define the projection $\Pi\gamma(0)$ over the fixed point set and the parrallel transport $\tau(\gamma(0),\Pi\gamma(0))$ from $\Pi\gamma(0)$ to $\gamma(0)$. Over M^g, we have the tangent bundle TM^g and its orthogonal bundle $(TM^g)^H$, which are parrallel because M^g is totally geodesic. We use the parrallel transport $\tau(x,\Pi x)$ in order to get a bundle T_gM and a bundle T_gM^H over a small tubular neighborhood of the fixed point set M^g. Moreover, T_gM and T_gM^H are orthogonal. If $\gamma(0)$ is in the small neighborhood of the fixed point set, we can split T_γ^0 in $T_\gamma^0(T_gM)$ and $T_\gamma^0(T_gM^H)$. This decomposition is orthogonal. We keep the Hilbert structure in $T_\gamma^0(T_gM)$ and, in $T_\gamma^0(T_gM^H)$, we take the Hilbert strucure as ($((1-\varepsilon^2)\, f((\gamma(0))/\varepsilon^2) + 1) = f_\varepsilon(\gamma(0))$ multiple of the Hilbert structure from the previous part : moreover $f(\gamma(0))$ is smooth ≥ 0, depends only from the distance between the starting point and the fixed point set M^g, and is equals to 0 outside a small tubular neighborhood U_1 of the fixed point set and is equals to 1 inside a smaller tubular neighborhood U_2 of the fixed point set. For the limit theorem we will do later, only the contribution of a small neighborhood of the fixed point will be significent : an orthonormal basis of $T_\gamma^0(T_gM)$ isthe same and an orthonormal basis of $T_\gamma^0(T_gM^H)$ is rescaled by ε. We won't write later all the details which comes from the fact that this rescaling is only true in fact over a small tubular neighborhood of M^g, by doing a suitable partition of unity associated to a neighborhood of the fixed point set invariant under the geometrical action of h.

These definition being given, we define the operator $d_{\varepsilon,r,g}$, the operator $d^*_{\varepsilon,r,g}$, the symmetric operator $d_{\varepsilon,r,g} + d^*_{\varepsilon,r,g}$ and the operator $(d_{\varepsilon,r,g} + d^*_{\varepsilon,r,g})^2 = \Delta_{\varepsilon,r,g}$ as before. Moreover since we choose $f_\varepsilon(x)$ depending only from the distance from x to M^g, and since that distance is invariant under the action of h, because h and g commute, all these operators can be chosen invariant under the action of h : the main difficulty is to show that the splitting into T_gM and T_gM^H is invariant under the action of h. But since h and g commute, h keeps M^g and therefore dh keeps the decomposition over M^g of TM in TM^g and $(TM^g)^H$. Moreover $\Pi h\gamma(0) = h\Pi\gamma(0)$, always because h and g commute. Moreover, let e_0 be a section of TM^g. $\tau(\gamma(0),\Pi\gamma(0))\, e_0(\Pi\gamma(0))$ is a section of T_gM.

We have :

$$dh\, \tau_t\{(1-t)\, \tau(\gamma(0),\Pi\gamma(0))\, e_0(\Pi\gamma(0)) + t\, \tau_1^{-1} dg\tau(\gamma(0),\Pi(\gamma(0))\, e_0(\Pi\gamma(0))\} = \tau_t(h\gamma)\{(1-t)\, dh$$

$$\tau(\gamma(0),\Pi\gamma(0))\, e_0(\Pi\gamma(0)) + t dh\, \tau_1^{-1}(dh)^{-1} d(hgh^{-1}) dh\, \tau(\gamma(0),\Pi\gamma(0))\, (dh)^{-1}\, dh\, e_0(\Pi\gamma(0))\} \tag{2.13}$$

$$=$$

$$\tau_t(h\gamma)\{(1-t)\, \tau(h\gamma(0),\Pi h\gamma(0))\, dh\, e_0(\Pi\gamma(0)) + t\, (\tau_1(h\gamma))^{-1} dg\tau(h\gamma(0),\Pi h\gamma(0))\, dh\, e_0(\Pi\gamma(0))$$

$$\}$$

and dh $e_0(\Pi\gamma(0))$ is a vector in $\Pi h\gamma(o)$ tangent to M^g. This shows that our splitting is kept near M^g.

We follow the line of [J.L$_2$] in order to define the Bismut's dilatation. We have our basis of distinguished vector fields X(n,e(i)) for a local smooth section e(i) of orthonormal basis. Moreover, over our little neighborhood of the fixed point set, we choose that local section with respect to the splitting of TM in T_gM and T_gM^H. We deduce from this an orthonormal basis X(I) of the fiber of differential forms. Moreover this choice is invariant under the action of h, because h keeps the splitting. Let us choose the coordinate of X(I). If $d(\gamma(0),M^g)$ is small, we take any finite sum of products of the type $f(\Pi\gamma(0))\ \Pi_{I(n)}\ (f_i(\gamma(t(i)) - f_i(\Pi\gamma(0)) = F$. I(n) is a finite part of cardinal n of [0,1]. Moreover all the I(n) with the same cardinal are distincts. Moreover if I(n) = t(1) < t(2)...<t(n), we suppose that the union of all the I(n) for n fixed is dense in the simplex t(1) < t(2)..<t(n) of $[0,1]^n$. If F is such a functional, $F(h\gamma)$ is still such a functional because $h(\Pi\gamma(0)) = \Pi(h\gamma(0))$, which shows us that the choice of such test functionals is invariant under the action of h, if $d(\gamma(0),M^g)$ is small. If $d(\gamma(0), M^g)$ is big, we take any cylindrical functional (We don't write completely the details about this, but we stick together the two componentsby using as test functionals $h(\gamma(0))\ F(\gamma) + (1-h(\gamma(0))\ G(\gamma)$ where h is a smooth function with compact support in a small neighborhood invariant under the action of h and equals to 1 in a smaller neighborhood invariant under the action of h. We perform the Bismut's dilatation only over the first component.).Let us suppose that $\Sigma_I\ f_{0,I}(\Pi\gamma(0))\ \Pi_I\ (f_{i,I}(\gamma(t_i)) - f_{i,I}(\Pi\gamma(0)))$ is equals to 0. Since all the I are distincts, we deduce that each $f_{0,I}(\Pi\gamma(0))\ \Pi\ (f_{i,I}(\gamma(t_i)) - f_{i,I}(\Pi\gamma(0))$ is equal to 0. We can now define the Bismut's dilatation over a functional $F = \Sigma\ f_{0,I}\ \Pi\ (f_{i,I}(\gamma(t_i)) - f_{i,I}(\Pi\gamma(0))\ )$ by putting :

(2.14) $B_\varepsilon F = \Sigma\ f_{0,I}(\Pi\gamma(0))\ \Pi\ (f_{i,I}(\gamma(t_i)) - f_{i,I}(\Pi\gamma(0)))/\varepsilon$

If $d(\gamma(0), M^g)$ is big, we don't change the functional. We have that key property, since $\Pi h\gamma(0) = h\Pi\gamma(0)$:

(2.15) $B_\varepsilon(F(h\gamma)) = (B_\varepsilon F)(h\gamma)$

Let us show that the space of scalar functionals where the Bismut's dilatation is defined is dense. This follows from this :

$$f(\Pi\gamma(0))\ \Pi_{I(n)}(f_i(\gamma(t_i))- f_i(\Pi\gamma(0)) = f(\Pi\gamma(0))(\ \Pi_{I(n-1)}\ (f_i(\gamma(t_i)) - f(\Pi\gamma(0))))\ f_n(\gamma(t_n))$$

(2.16)

$$-f(\Pi\gamma(0))\, f_n(\Pi\gamma(0))\, \Pi_{I(n-1)}(f_i(\gamma(t(i))) - f_i(\Pi\gamma(0)))$$

By induction over n, we suppose that each functional $f(\Pi\gamma(0),\gamma(t(1),..,\gamma(t(n-1))$ is limit of sum of finite products with the cardinal of I(k) smaller of n-1. If we use this induction hypothesis, it results from (2.16) that we can get any functional of the type $f(\Pi\gamma(0),\gamma(t(1),..,\gamma(t(n-1))\, f_n(\gamma(t(n)))$ in $L^2(\mu_g)$, and therefore all the functionals which are in $L^2(\mu_g)$ by the Stone-Weierstass theorem.

Let us define the Bismut's dilatation for forms : we choose an orthonormal basis $e_i(\Pi\gamma(0))$ of T_gM and an orthonormal basis $e_i(\Pi\gamma(0))$ of T_gM^H. We deduce a basis X(I) of our fiber of differential forms. If we change of orthonormal basis $e_i(\Pi\gamma(0))$, the change of basis X(I) is seen only by terms which depend only on $\Pi\gamma(0)$. If $\sigma = \Sigma F_I X_I$, let us define

(2.17) $B_\varepsilon\sigma = \Sigma (B_\varepsilon F_I) X(I)$

This definition is coherent from the remark before. If $d(\gamma(0), M^g)$ is big, there is no operation, and we stick in a smooth way these two operations, but it does not give difficulties, because when ε tends to 0, only the contribution of the small tubular neighborhood of M^g appears.

We have still the basical property :

(2.18) $B_\varepsilon(dh\sigma) = dh\,(B_\varepsilon\sigma)$

Let us now define the limit model, conformally to [J.L$_2$] and [T].

The probability space is defined as follow:

-)Over M^g we take the bundle of bridge in TM which goes from c to dg c, c being in $(TM^g)^H$. Over M^g, we put the Riemannian measure and over the set of path which go from c to dgc in time 1, we put the measure $\exp(- \|(I-dg)c\|^2)\, dc \otimes P_{1,c,dgc}$ is the law of the Brownian bridge in T_xM (and not in $(T_xM^g)^H$) which go to c to dgc. Let us recall that the Brownian bridge which go to c to dgc has the same law as the process $(1-s)c + s\, dg\, c + \gamma_{s,flat}$ is a flat Brownian bridge starting from 0 and coming back to 0 in T_xM in time 1. The introduction of this model is motivated by [Bi$_4$].

As tangent space of the flat Brownian bridge $\gamma_{s,flat}$, we take the space H of finite energy element h of T_xM such that h(0)=h(1)=0 with the Hilbert norm $\int_{[0,1]}\|h'(s)\|^2_x\, ds$. Over the set of c, we take the Hilbert norm $\|c\|^2$. The fact we use the Hilbert norm $\|c\|^2$ instead of the norm vector field $X(0,e(i))$. Over an element of that probability limit space, we get as fiber $\Lambda_x \wedge \Lambda_c \wedge \Lambda_{fermionic}$. The last exterior algebra is the fermionic Fock space associated to the flat Brownian bridge starting from 0 in T_xM.

As limit operator, we choose :

$$d_{x,g} + d_{c,g} + d_{\infty,g} = d_{x,g} + d_{2,1,g} = d_{1,g} = \sum A(0)\, e(i) \wedge \Gamma_{e(i)} + \sum A(0)\, c(i) \wedge \Gamma_{e(i)} +$$

(2.19)

$$\sum A(n)(\cos(ns)\, e(j)) \wedge \Gamma_{\cos(ns)\, e(j)} + \sum A(n)\, (\sin(ns)\, e(j)) \wedge \Gamma_{\sin(ns)\, e(j)}$$

In the first sum, we take derivative over an orthonormal basis $e(i)$ of T_xM^g (x belongs to M^g). In the second sum, we take derivative over an orthonormal basis $e(i)$ of $(T_xM^g)^H$ in the limit Gaussian space. In the third sum, we take the classical Araï-Shigekawa complex corresponding to the $A(n)$ and to the Brownian bridge in the full tangent space of M in x starting from 0 and coming back in 0 in time 1. d_x, d_c, d_∞ anticommute as it can be seen in normal coordinates. If we work in normal coordinates, we can compute the adjoint of d_1. $d^*_{1,g}$ is given

$$d^*_x + d^*_c + d^*_\infty = -A(0) \sum i_{e(i)} \Gamma_{e(i)} - A(0) \sum i_{c(i)} \Gamma_{c(i)} - \sum A(n)\, i_{\cos(ns)\, e(j)} \Gamma_{\cos(ns)\, e(j)}$$

(2.20)

$$-\sum A(n) i_{\sin(ns) e(j)} \Gamma_{\sin(ns) e(j)} - A(0) \sum i_{c(i)} <(1-dg)c,(1-dg)c(i)> - \sum \int_{[0,1]} <\cos(ns)\, e(j), \delta\gamma_{flat,s}>$$

$$- \sum A(n) <\sin(ns)\, e(j), \delta\gamma_{flat,s}>$$

It is the same type of formulas as in [J.L$_2$], but the normal flat Brownian bridge is more complicated here, because we choose too c in random. If we put c and γ_{flat} together, we have an abstract Wiener space, and $d_{c,g} + d_{\infty,g}$ can be understood in the formalism of Araï [Ar$_1$], [Ar$_2$], [Ar.M]. We can choose namely the $c(i)$ such that it is an orthonormal basis for $(TM^g)^H$ for the norm $\| c \|^2$. Let us recall namely that over M^g, if we write dg as a collection of matrices of rotation of angle θ, we get a collection of orthogonal subbundles which are parrallel over each component of M^g. Modulo this $d^*_c + d^*_\infty$ appears as an Araï operator with an auxiliary operator in c for the Gaussian space is spanned by c and the flat Brownian bridge γ. As Fermionic Fock space, we choose $\Lambda_c \wedge \Lambda_\gamma$ with the norm $\| c \|^2$ and as Bosonic Fock space, the space L^2

associated to the limit Gaussian probability measure $\exp(-\|(1-dg)c\|^2)\otimes dP_{1,x}$. The auxiliary operator in c is the operator which allows us to pass from the both Hilbert strcture in c.

Moreover $d_{\infty,g} d^*_{\infty,g} + d^*_{\infty,g} d_{\infty,g} = \Delta_{\infty,g} = N_B(A^2) + N_F(A^2)$ and $d_{c,g}d^*_{c,g} + d^*_{c,g}d_{c,g} = \Delta_{c,g} = N_B(c^2) + N_F(c^2)$. The number operator for boson $N_B(A^2)$ is associated to the operator which sends sin(ns) associates $A(n)^2$ sin(ns)and cos(ns) to $A^2(n)$ cos(ns)as well as the fermion number operator $N_F(A^2)$. The bosonic number operatore $N_B(c^2)$ and the fermionic number operators $N_F(c^2)$ are related to the change of Hilbert structure in $(TM^g)^H$.

Therefore, $d_{1,g} + d^*_{1,g}$ has a symmetric extension. Namely $\Delta_{c,g} + \Delta_{\infty,g}$ can be diagonalised, because it is a sum of bosonic number operators and of fermionic operators (See [Ar_2]). Since the A(n) don't depend on x and since the diagonalisation of dg is parrallel over M^g, we deduce that the set of eigenvectors associated to different eigenvalues of $\Delta_{c,g}$ and of $\Delta_{\infty,g}$ constitue a countable set of finite dimensional bundle over M^g, which are preserved by $d_{x,g} + d^*_{x,g}$, $d_{c,g}+d_{c,g}$ and $d_{\infty,g} + d^*_{\infty,g}$, this because these operators are anticommuting (cf[T] and [JL_2]). $d_{x,g} + d^*_{x,g}$ appears exactly over each bundle as the de Rham operators tensorised by this bundle. We know that the spectrum of $d_{x,g} + d^*_{x,g}$ is discrete over each of this finite dimensional bundle, as well as $d_{c,g} + d^*_{c,g}$ and $d^*_{\infty,g} + d_{\infty,g}$. Moreover the action of $d_{c,g}+d^*_{c,g}$ and of $d_{\infty,g} + d^*_{\infty,g}$ over each of these bundles is the square root of the action modulo the sign of $\Delta_{c,g}$ and of $\Delta_{\infty,g}$ over each of this bundle. This allows us by restricting these bundles to diagonalise $d_{1,g} + d^*_{1,g}$ and to show it has a self-adjoint extension.

We can look at the action of h over the limit model. h keeps M^g because g and h commute. Moreover dh lifts over M^g to a natural action over TM, which preserves TM_g and $(TM_g)^H$. Let us remark that :

(2.21) $\| dh(1-dg)c\|^2 = \| (1-dg)(dhc)\|^2$

such that the action of dh preserves the auxiliary operator which appears in $\Delta_{c,g}$ and $\Delta_{\infty,g}$(since the action of dh preserves the metric of the tangent space of γ_{flat}). This shows us that dh commute with all the limit operators given before.

THEOREM II 1 : *If* A(n)> | n |^p

gh = hg

(2.22) $\mathrm{Tr}\exp(-t\Delta_{1,g}h) < \infty$

(2.23) $\mathrm{Ind}\,(d_{1,g} + d^*_{1,g})\,h = \chi(M^g \cap M^h)$

Proof : The proof of the existence of the trace follows directly the line of [J.L2], because h keeps the Wick product and the fermionic Fock space.Let Ξ_K be a such a subbundle for $\Delta_{c,g}$ and $\Delta_{\infty,g}$ endowed with a given combination of Wick product in sin(ns,) cos(ns,) and of exterior algebra in cos(ns) and sin(ns). K denotes the combination of sin(ns),cos(ns) which appear in Ξ_K : it is possible there is more one than one of each sin(ns) which appears there. $|K|$ is the cardinal of K. The dimension of such subbundle is bounded by $C^{|K|+1}$,and the action of $\exp(-t\Delta_{2,1,g})$ over each subbundle is diagonal and bounded by $C\exp(-t\Sigma_K \Lambda(n)^2)$. The action of Δ_x over $\Lambda_x \wedge \Xi_K$ is given by the Lichnerowicz formula $\Delta_K = -1/2\Delta^{M^g} + C_1 + C_K$. C_1 is the action of the Lichnerowicz formula for the non-tensorised de Rham operator, and C_K comes from the action of the Lichnerowicz formula over the auxiliary bundle, which appears as a combination of at most $|K|$ products of 3 types together: exterior products, symmetric tensor products and tensors products. We have a probabilistic representation of the trace of the heat semi-group associated to Δ_K , since over each product we take the connection product. Let $\tau_{s,K}$ be the parrallel transport over Ξ_K, which preserves the product, and let

(2.24) $dU_{s,K} = -1/2\, U_{s,K}\, \tau_{s,K}^{-1}\, (C_K + C_1)$

We get the following representation of [Bi3],[I.W], [Le2],[Le3] and more precisely [Bi4] of the trace of the heat semi-group :

(2.25) $\mathrm{Tr}\exp(-t\Delta_K)h = \int_{M^g} p_t(x,hx)\, E_{t,x,hx}(\mathrm{tr}\,(U_{1,K}\, \tau_{1,K}^{-1}\, dh))\,dx$

where $p_t(x,y)$ is the heat-kernel associated to the Brownian motion over M^g and $E_{t,x,h(x)}$ the expectation for the Brownian bridge which goes from h(x) to x in time t. In particular, we have a bound of the trace under the expectation in $C\; C^{(1+t)|K|}\; \Pi_K \exp(-t|A(n)|^2 < \Sigma C^{|K|(1+t)}\, \Pi_K \exp(-\tau n^2 P) = C\, \Pi_K\, C^{(1+\tau)} \exp(-t\, n^2 P) < \infty$.

This shows that the first part of the theorem is true.

Let us shows now that the second part of the theorem is true. The operators $d_{\infty,g}$, $d_{c,g}$, $d_{x,g}$ anticommute or commute with h. If a section belongs to the kernel of $d_{1,g} + d^*_{1,g}$, it is therefore by using Araï's computation [Ar1], [Ar2] a form in x which does not depend from γ_{flat} and c, almost surely. This shows us that

(2.26) $\mathrm{Ind}_h (d_{1,g} + d^*_{1,g}) = \chi_h(M^g)$

We apply the classical Lefschetz theorem and we find

(2.27) $\chi_h(M^g) = \chi(M^g \cap M^h)$

since h is an isometry of M^g because g and h commute.

Let us now motivate the introduction of these operators by the following limit theorem which is analoguous to the limit theorem of [J.L2]. But before this, we need to understand what we need by a limit in law, because our situation is a little bit more complicated than the situation encoutered in [J.L2]. Let us recall that the fiber is isomorphic to $\Lambda(T_xM) \wedge \Lambda_x(H)$. But if x is close to the fixed point set, $\Lambda(T_x) \wedge \Lambda_x(H)$ is isomorphic by means of the parrallel transport between x and $\prod x$ to $\Lambda(T_{\prod x}) \wedge \Lambda_{\prod x}(H)$. We identify the fiber closely to M^g with $\Lambda(T_{\prod x}) \wedge \Lambda_x(H)$ and far to M^g to the original fiber. We put as Hilbert space structure the space of L^2 section over $\Lambda(T_{\prod x}) \wedge \Lambda_{\prod x}(H)$ and the space of L^2 section over $\Lambda(T_x) \wedge \Lambda_x(H)$ far from our neighborhood. An L^2 section of form over the twisted loop space appears therefore as a L^2 random variable from the twisted loop space into this fixed Hilbert space. It has sense in particular to speak of the limit in law of such random variable into this fixed Hilbert space which justifies our conjecture.

THEOREM II 2 :*For any fixed element of* $\mathring{A}_g$, *we have in law if* $4\rho < 1$:

(2.29) $B_\varepsilon \sigma \to \sigma_1$

(2.30) $B_\varepsilon\, dh\sigma \to dh\, \sigma_1$

(2.31) $(d_{\varepsilon,r,g} + d^*_{\varepsilon,r,g})\, dh\, B_\varepsilon\sigma \to (d_{1,g} + d^*_{1,g})\, dh\, \sigma_1$

(2.32) $\Delta_{\varepsilon,r,g}\, dh\, B_\varepsilon\sigma \rightarrow \Delta_{1,g}\, dh\, \sigma_1$

Proof: Let us begin to show first that $B_\varepsilon\sigma \rightarrow \sigma_1$ in law for any element of $\check{A}_g$. This arises from the Bismut's computation of [Bi4] : any finite combination of $(f(\gamma(t)) - f(\prod\gamma(0)))/\varepsilon$ tends in law to $<df(\gamma(0)),\gamma_{flat}(t) + (1-t)e + t\, dg\, e>$ for the given limit probability Gaussian measure. The second affirmation comes from the fact that $dh\sigma$ belongs to $\check{A}_g$ and that $B_\varepsilon\, dh\, \sigma = dh\, B_\varepsilon\sigma$. Let us remark that we don't need the full Bismut procedure in order to see that, because we take only a finite number of terms in $(f(\gamma(t))-f(\prod\gamma(0)))/\varepsilon$. Computations similar to [Le2] can be used.

Let us show now that $(d_{\varepsilon,r,g} + d^*_{\varepsilon,r,g})\, B_\varepsilon\sigma$ tends in law to $(d_{1,g} + d^*_{1,g})\, \sigma_1$ (We can remove the term in h, this from (2.30)). We work in normal coordinates in $\prod\gamma(0)$. Let us begin by the divergence part in $d^*_{\varepsilon,r,g}$. If we take an element of T^n_γ, $n > 0$, of the distinguished basis, it is multiplied by ε, this from the rescaling of the metric. We get :

$$\operatorname{div}\varepsilon X(n,e(i)) = \varepsilon/\varepsilon^2\int_{[0,1]} < \tau_s \cos(ns)\, e(i),\delta\gamma(s) > + 1/2\, \varepsilon^2/\varepsilon^2 \int_{[0,1]} < S_{\varepsilon X(n,e(i))(s)},\delta\gamma(s) > + \tag{2.33}$$

counterterms.

The counterterms disappear when ε tends to zero, and in law it remains at the end $\int_{[0,1]}< \cos(ns)\, e(i),\delta\gamma_{flat}(s) >$ which is exactly the divergent term which appears in $d^*_{\infty,g}$. In the limit contribution, there is no term in $\int_{[0,1]} <\cos(ns)\, e(i), (1-dg)e ds > = 0$. The case $n < 0$ is similar. For the moment, we don't see the difference with the computation of [J.L2].

The difference appears namely when we want to treat the contribution in the divergent part of T_γ^0, because there is in this case two distinct behaviour.

Let us consider first the case of $X_0(\tau(\gamma(0)),\Pi\gamma(0))\, e(i)(\prod\gamma(0))$ where $e(i)$ is orthogonal to TM^g. In this case, the metric is rescaled, and we have to multiply near M^g our vector by ε. We get :

$$\varepsilon\operatorname{div}X(0,e(i)) = \varepsilon\operatorname{div}\tau(\gamma(0),\Pi\gamma(0))\, e(i)(\prod\gamma(0)) + \varepsilon/\varepsilon^2 \int_{[0,1]} <\tau_s(-\tau(\gamma(0),\Pi\gamma(0))e(i)(\Pi\gamma(0)) + \tau_1^{-1}\, dg\, \tau(\gamma(0),\Pi\gamma(0))e(i)(\prod\gamma(0))),\delta\gamma(s) > + \varepsilon^2/\varepsilon^2 \int_{[0,1]} <S_{\varepsilon X(0,e(i))},\delta\gamma(\sigma)> + \text{counterterm} \tag{2.34}$$

In this case, the limit in law of a finite family of divergence of this kind is

<(-1+dg)e(i)(γ(0)),(-1+dg)e > which gives the divergence part of the term in $d^*_{c,g}$ in the limit model.

If we work now over TM^g, we have the same type of behaviour, but this goes in law to div e(i), because τ_1 has a behaviour in $1+\varepsilon^2$ and because e(i)(Πγ(0)) belongs to the kernel of -1+dg. This shows us in local coordinates that the divergence part of $d^*_{\varepsilon,r,g} B_\varepsilon\sigma$ converges to the divergence part of $d^*_{1,g}\sigma_1$, and this without the Bismut's procedure, because in this case we have only finite expression. (In fact, it is not so simple, because in limit theorem in law, we take test functional which are only continuous, and it is not easy to regularised continuous test functions in the non compact case.In order to be rigorous, we cannot avoid to use the Bismut's procedure. See later for this).

Let us now study the behaviour of $d_{\varepsilon,r}B_\varepsilon\sigma$. The difficulty is now that we have infinite expressions. If n > 0, we have to study the behaviour of < d(f(γ(t))−f(Πγ(0)))/ε, εX(n,e(i))> which is equals to <df(γ(t)),X(n,e(i))(t)> because X(n,e(i))(0) = 0. This tends to $<df(\gamma(0)),\int_{[0,t]}\cos(ns)ds\ e(i)>$, which is exactly the derivative of $<df(\gamma(0)),\int_{[0,t]}\delta\gamma_{flat}(s)>$ in the direction cosns e(i) of the Cameron-Martin space H of the Brownian bridge. The case n<0 is similar. If we take a derivative in the direction of $(TM^g)^H$, Πγ(0) does not change asymptotically in ε under the action of such vector field. The vector field is rescaled by ε itself, because we rescale the metric in this direction. So we find that in law <d(f(γ(t))) - f(Πγ(o))),X(o,e(i))> tends to <df(γ(0)),t(-1+dg)e(i)> which is exactly the derivative of <df(γ(o)),t(-1+dg)e> in the direction e(i). Let us now study the behaviour in law of the derivatives in the direction of TM^g.We get if e(i) belongs to TM^g

$$<d(f(\gamma(t)-f(\Pi\gamma(0)))/\varepsilon,\ X(0,(e(i))> \ = \ <d(f(\gamma(t)-f(\gamma(0)),X(0,e(i))>/\varepsilon\ +$$

(2.35)

$$<d(f(\gamma(0)-f(\Pi\gamma(0)),X(0,e(i))>/\varepsilon$$

Since we work in normal coordinates, and since $\tau_1(\gamma(0),\Pi\gamma(0)) \approx 1+\varepsilon^2$ when ε tends to 0, the derivative of the second term disappears almost completely when ε goes to 0, because d(γ(0),Πγ(0)) ≈ ε when ε goes to 0 : it remains at the limit $<\Gamma_{e(i)}df(\gamma(o)),\ e>$. Let us treat the first term. But it is in the Stratonovitch sense :

$$<d\int_{[0,t]}<df(\gamma(s),d\gamma(s)>,\ X(0,e(i)>/\varepsilon\ =\ \int_{[0,t]}<\Gamma_{X(0,e(i))(s)}df(\gamma(s)),d\gamma(s)>/\varepsilon$$

(2.36)

$+ \int_{[0,t]} < df(\gamma(s), \tau_s(-\tau\ (\gamma(0), \Pi\gamma(0))\ e(i)(\Pi\gamma(0)) + \tau_1^{-1}\ dg\tau\ (\gamma(0), \Pi(\gamma(0))\ e(i)(\Pi\gamma(0))>/\varepsilon$

The second term tends to 0 because $\tau_1^{-1}\ dg = I + \varepsilon^2$ in law and at the end we see that :

$$<d(f(\gamma(t) - f(\Pi\gamma(0)), X(0,e(i))>/\ \varepsilon \to \int_{[0,t]} < \Gamma_{e(i)} df(\Pi\gamma(0)), \delta\gamma_{flat,s} + (-1+dg)cds > +$$

(2.37)

$$< \Gamma_{e(i)} df(\Pi\gamma(0)), c>.$$

Let us remark that the lot of simplification which appears because we use local normal coordinates over $\Pi\gamma(0)$ (for instance $\tau_1 \approx 1 + \varepsilon^2$) appears in general not in each part of the operator but only globaly. For instance, the derivatives of the distinguished vector fields cancel when ε tends to 0 because we use that normal coordinate system in $\Pi\gamma(0)$: without this the computation should be more complicated. The difficulty we have to overcome is that we get in fact an infinite sum. In order to solve that, we will use the Bismut's procedure as given in [Bi_2],[Bi_4] and not in [Le_1], because in this case some smoothness assumption is necessary about the auxiliary functional of the Brownian bridge which is considered. Let us consider the collection of $<df_i(\gamma(t)), \tau_t \int_{[0,t]} \cos(ns)\ ds\ e(j) >$, $<df_i(\gamma(t)), \tau_t \int_{[0,t]} \sin(ns)\ ds\ e(j)>$. It is a random element Φ_ε of $L^2(\mathbb{N})$. We have to show that for all bounded continuous functionals F from $L^2(\mathbb{N})$ into $\mathbb{R}$, the expectation of $F(\Phi_\varepsilon)$ tends to the expectation of $F(\Phi)$. We use the Bismut's fact that :

$$(2.38)\quad \mu_{\varepsilon,g}\,(\,F(\Phi_\varepsilon(\gamma))) = \mu(\,F\Phi(x, \varepsilon\gamma_{flat} + \varepsilon(\,(1-s)c + dgsc) + \varepsilon\, v^2(\varepsilon\gamma,c,x)\,)) + o(\varepsilon^2)$$

In order to get v^2, we look the equation

$$(2.39)\quad du_s = \sum X_i(u_s)\,(\varepsilon d\gamma_{s,flat} + \varepsilon(-1+dg)c\ ds + \varepsilon v^2 ds)$$

where the X_i are the canonical vector fields over the frame bundle over M. We suppose that u_s start from $\Pi u(o) = \gamma(0) + \varepsilon c$, $\gamma(0)$ belonging to M^g and c being in TM^g, this expression being written in a tubular neighborhood. We choose v^2 such that $\gamma(1) = \Pi u_1$ is equal to $\gamma(0) + \varepsilon dgc$. We start from $\gamma(0) + \varepsilon c$, because the heat kernel $p_{\varepsilon}2(x,gx)$ does not tend to zero when x has a behaviour in $\gamma(0)+\varepsilon c$ (See [Bi_4] for more details). The key fact is the following : generally for a given ε, we cannot find a v^2 such that $\gamma(1) = \gamma(0) + \varepsilon dgc$, but it is asymptotically true, and this uniquely (See [Le_1] for a non-geometrical approach). This explains the error term in (2.38)as well it is explained by the contribution of the Jacobian which appears in the implicit function theorem which tends to 1 when ε goes to 0. The last difficulty it remains to explain is that there is τ_t which is not a continuous functional of γ which appears. But it is overcomed because τ_t in (2.39) appears (It works too if we take a finite number of stochastic integrals with different integrands).

This type of argument works too (but in a simpler way, because there is in this case a finite sum) for the non divergent part of $d^*_{\varepsilon,r,g}\ B_\varepsilon\sigma$. In conclusion, we have shown that in law $(d_{\varepsilon,r,g}+ d^*_{\varepsilon,r,g})\ B_\varepsilon\sigma$ tends to $(d_{1,g}+d^*_{1,g})\ \sigma_1$.

Let us show now that $(d_{\varepsilon,r,g} + d^*_{\varepsilon,r,g})^2\ B_\varepsilon\sigma\ = \Delta_{\varepsilon,r,g}\ \sigma$ converges in law to $\Delta_{1,g}\sigma_1$.

a)Let us begin by the simplest contribution, that means $d^*_{\varepsilon,r,g}\ d^*_{\varepsilon,r,g}\ B_\varepsilon\sigma$. It is a finite sum which appears. Moreover since two interior product anticommute and since there the derivatives in the limit probability space over $\gamma(0)$ in M^g commute, the limit in law of this expression is nothing else than $d^*_{x,g}d^*_{x,g}\ \sigma_1$.

b)Let us look the contribution of $d_{\varepsilon,r,g}\ d_{\varepsilon,r,g}\ B_\varepsilon\sigma$. There is a doubly infinite sum in this expression. Since we look in normal coordinates, we see that the apparently most difficult part to handle in this expression is $\sum_{n\neq 0,m\neq 0,i,j}\ A(m)A(n)<d<dB_\varepsilon F_1,\varepsilon X(n,e(i))>,\varepsilon X(m,e(j))> X(m,e(j))\wedge X(n,e(i)) \wedge X(1)$.The contribution with n=m, e(i) = e(j) cancels. We have only to consider the family of $A(m)A(n)\ \ \varepsilon^2(\ <d<dB_\varepsilon F_1,X(n,e(i))>,X(m,e(j))> - <d<dB_\varepsilon F_1,X(m,e(j)>,X(n,e(i))>)$ which belongs to $L^2(\mathbb{N})$. The only difficulty is when we derive twice the same $(f(\gamma(t(i)))-f(\Pi\gamma(0)))/\varepsilon$: in the other case, there is a automatical cancellation. We use another time the Bismut's procedure, but we have to use the formula (2.7). An infinite number of stochastics integrals with different integrands appears . We overcome this difficulty by writtng $X_s = \tau_s\ H_s$ and by integrating by part in (2.7). The boring term are of the type $\tau_t\int_{[0,t]}K_sH'_sds$ where a fixed K_s (independent of H_s) appears. K_s is a Stratonovitch integral in the curvature tensor and τ_s : we can apply the Bismut's procedure to K_s, which allows to conclude. Let us remark that the fact that the second derivative of $f(\gamma(t_i))- f(\Pi\gamma(0))/\varepsilon$ along $\varepsilon X(n,e(i))$ and $\varepsilon X(m,e(j))$ cancels at the limit describes the fact that the derivative of $<df(\gamma(0))\gamma_{flat,t} + (1-t)\ c + t\ dg\ c>$ is deterministic at the limit in the direction of the tangent space of the Brownian bridge.

In this case, the computation was easier because we divide each $f(\gamma(t(i)))-f(\prod\gamma(o))$ by ε and we multiply each $X(n,e(i))$ by ε. This simplification does not appear when we have to multiply only one $X(n,e(i))$ by ε. We look the convergence in law of the series in $L^2(\mathbb{N})$ $A(n)\varepsilon\{ \langle d\langle dB_\varepsilon F_I, X(0,e(i))\rangle, X(n,e(j))\rangle - \langle d\langle dB_\varepsilon F_I, X(n,e(j))\rangle, X(0,e(i))\rangle\}$ where $e(i)$ belongs to TM^g, $n\neq 0$, or $e(j)$ belonging to $(TM^g)^H$, $(n=0.)$ Only the contribution of the second derivative of the same $(f(\gamma(t(i)))-f(\prod\gamma(0)))/\varepsilon$ plays a role. We have for $n\neq 0$

$$\langle d\langle d(f(\gamma(t(i)) - f(\gamma(0))), X(0,e(i))\rangle, X(n,e(j))\rangle A(n) =$$

(2.40)

$$\langle d\langle df(\gamma(t), X(0,e(i))(t)\rangle, X(n,e(j))\rangle A(n)$$

and

$$\langle d\langle d(f(\gamma(t(i)) - f(\prod\gamma(0)), X(n,e(j))\rangle, X(0,e(i))\rangle A(n) =$$

(2.41)

$$\langle\langle df(\gamma(t(i)), X(n,e(j))(t)\rangle, X(0,e(i))\rangle A(n)$$

We have to take the derivative $\Gamma_{X(0,e(i))}X(n,e(j))(t)$ and $\Gamma_{X(n,e(j))}X(0,e(i))(t)$. These two sequences tend separetely in law to 0, because we work in the normal coordinate system. Let us repeat that this simplification appears separately because we use normal coordinates over each contributor term of the operator appears globally over the operator, which is intrinsically defined. It remains only to study the contribution of the sequence :

(2.42) $A(n)\,(\langle\Gamma^2 f(\gamma(t)), X(0,e(i))(t), X(n,e(j))(t)\rangle - \langle\Gamma^2 f(\gamma(t)), X(n,e(j))(t), X(0,e(i))(t)\rangle)$

which tends in $L^2(\mathbb{N})$ in law to 0 because we work in normal coordinates. We have shown that $d_{\varepsilon,r,g}\, d_{\varepsilon,r,g}\, B_\varepsilon\sigma$ tends in law to $d_{x,g}d_{x,g}\sigma_I$.

c) We consider the case of $d_{\varepsilon,r,g}d^*_{\varepsilon,r,g}B_\varepsilon\sigma$. Since $d^*_{\varepsilon,r,g}B_\varepsilon\sigma$ is a finite sum, that term can be treated as the contribution of $d_{\varepsilon,r}B_\varepsilon\sigma$. The first difference is that we have to take derivatve of the parrallel transport, and therefore to use (2.7). We have too to take derivative of the divergence part. The only difficulty in this case is to take the derivative of $1/\varepsilon \int_{[0,1]}\langle\tau_s H'(n)(s), \delta\gamma(s)\rangle$. It is in fact too a Stratonovitch integral. We have then if $m\neq 0$:

$$< 1/\varepsilon \int_{[0,1]} <\tau_s H'(n)(s), d\gamma(s)>, \varepsilon X(m,e(i))> = \int_{[0,1]} <\tau_s H'(n)(s), \tau_s H'(m)(s)> ds$$

(2.43)

$$+ \int_{[0,1]} < \tau_s \int_{[0,s]} \tau_u^{-1} R(d\gamma_u, X(n,e(i))(u)\ \tau_u H'(n)(s), d\gamma(s)>$$

which tends in law to $\int_{[0,1]} <H'(n)(s),H'(m)(s)> ds$, therefore the derivative of the divergence $\int_{[0,1]} <H'(n)(s),\delta\gamma_{flat}(s)>$ into the other direction $H(m)(s)$. The fact that the second term tends in law to zero does not come from the fact we use normal coordinates, but we have to use this for the derivatives of the others parts of the divergence.

d)The most complicated term to treat is $d^*_{\varepsilon,r,g} d_{\varepsilon,r,g} B_\varepsilon \sigma$ because the sum in $d_{\varepsilon,r,g} B_\varepsilon \sigma$ is infinite,among which is the term :

$$\sum_{n\neq 0,i} A(n)^2 \varepsilon^2 <d<dB_\varepsilon F_I, X(n,e(i))>, X(n,e(i))> - \varepsilon^2 \sum A(n)^2 <dB_\varepsilon F_I, X(n,e(i))>$$

(2.44)

$$\mathrm{div} X(n,e(i))$$

The first term does not put any problem, because each term of the seriesis in L^2 bounded by $CA(n)^2/n^2$ and since $2\rho < 1$. For the second, the most complicated term is

(2.45) $D(\varepsilon) = 1/\varepsilon \sum_{n\neq 0,e(i)} A(n)^2 <df(\gamma(t)), \tau_t H(n)(t)\, e(i)> \int_{[0,1]} <\tau_s H'(n)(s)e(i), \delta\gamma(s)>$

The deterministic serie $A(n)^2 H_n(t)$ is in $L^2(N)$ because $4\rho < 1$. Let $\phi_t(s) = \sum A(n)^2 H(n)(t) H'(n)(s)$. It is an element deterministic of $L^2[0,1]$, which does not depend on ε. We recognise in (2.45)

(2.46) $D(\varepsilon) = 1/\varepsilon \sum <df(\gamma(t)), \tau_t \int_{[0,1]} < \tau_s \phi'_t(s)e(i), \delta\gamma(s)>>$

Since $\phi'_t(s)$ is deterministic, this converges in law too $\sum <df(\gamma(0)), \int_{[0,1]} < \tau_s \phi'_t(s)e(i), \delta\gamma(s)>>$. Since $\phi'_t(s)$ is deterministic, this converges in law too $\sum <df(\gamma(0)), \int_{[0,1]} <\phi'_t(s), \delta\gamma_{flat}(s)>$ which is the divergent part of the operator associated to the auxiliary operator which to $H'(n)$ associates $A(n)^2 H'(n)$ over the flat Brownian bridge.

Remark : We separate in order to give a nice exposure of the convergence in law of different part of the considered expression, although it is not completely correct. But the convergence in law for all the expression together is ensured.

Remark: The theorem II.2 justifies the name of limit model, although we omit to speak about the difficulties of this limit procedure: it is perhaps possible to define another set of functionals such that the Bismut dilatation gives another limit operator.

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